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# THE MATHEMATICS TEACHER

*Devoted to the interests of mathematics in Elementary and Secondary Schools*


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# THE MATHEMATICS TEACHER

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William Betz

President of the National Council of Teachers of Mathematics

THE MATHEMATICS TEACHER

Volume XXV

Number 5

Edited by William David Reeve

# What Mathematics Means to the World*

*By* E. R. HEDRICK

*University of California at Los Angeles*

*1. Introduction.*—The title that I have chosen—"What Mathematics Means to the World"—has an intentional double meaning that will be evident to you.

What mathematics means to many is but the sarcastic exaggeration of painful memories of childhood. It seems to me sometimes, when I hear remarks by certain educational orators, that this must be precisely the motive for their viciousness, and that they have deliberately shut their minds to any consideration of its real or possible values. The unthinking public is less blamable for such an attitude; theirs is one of superficial and devil-may-care condemnation of all that is *high-brow*, that is, all that pretends to be serious, without deliberate intention to do either good or harm. The spirit of flaming youth has permeated too far the thinking of all ages of our population, who have seen in the past few years only easy money for all who flaunt serious endeavor, and who now clamor for some savior to bring back to them the golden age of 1929 without perceptible effort or sacrifice upon their own parts. In such a maddened world, torn by successive emo-

* Address delivered at the annual banquet of the National Council of Teachers of Mathematics at the Raleigh Hotel in Washington, D.C., on Saturday evening, February 20, 1932. Reprints of this article may be had from THE MATHEMATICS TEACHER for 15¢ each, postpaid.

tions of getting-rich-quick and of restoring a lost El Dorado, a contemplative subject has no immediate appeal, nor do they even stop to consider that the answers to their present perplexities rest upon a mathematical foundation. To this world, in its mad race for speed and gold, the only memory of things mathematical is vague and distorted: perhaps they try to think, Alice-in-Wonderland fashion, and produce this gem of thought, that the shortest distance between two points is a straight eight.

What does the World think of us? Candidly, that we are a lot of old fogies blundering around with worn-out lore of the past, knowing naught of their gilded paradise which just at present needs only to be re-gilded by the billions from the national treasury to make them sing again the jazz that masks the shallowness of our age. To toil not nor to spin appears to many as a biblical injunction. A comprehension of the far-reaching effects of mathematics on our civilization and on the pressing problems of the day would be to them abhorrent, as it seems to be for other reasons to those educational orators to whom I have already paid my respects. Comprehension of the purpose of it all was illustrated to me recently by a story of a little girl who, with her mother, watched an inexperienced golfer trying to drive off from his tee without success; finally he stopped for breath, and the little girl said to her mother: "Well, mamma, I guess he must have killed it, he isn't hitting at it any more." Mathematics is not dead, despite the driving of the oratorical enemies.

What *might* the World think of us? What are the relations of mathematics to our civilization? To the pressing problems of our day? To life? To the struggling millions who now look so desperately for aid to life? What is the rôle of mathematics in this modern world? Can they still comprehend?

Upon us, teachers of mathematics, rests the responsibility for what the World thinks of mathematics. The present apathy, the present ignorance, the present bitter opposition to mathematics as a school subject, the present misconceptions, are but the results of a faulty presentation of our subjects to our students. Yes, we have had the chance. And what have we done with it? When I speak of usefulness, some of my friends will not hear me, and they insist upon the beauties of mathematics, its cultural worth. To these I am not blind. But if we have not dwelt upon the usefulness of mathematics because we wished to show its beauties and its cultural worth, where are among the

millions who have studied mathematics in the schools those who have been impressed by its beauties and by its cultural worth? Why are they not rising in defense of these noble traits, which mathematics certainly possesses? Has our teaching failed also in that?

The World in which we live is a forgetful World. It treasures, all too much, crass pleasures of the moment, still crasser symbols of success whose measure is in money rather than in the spirit. Pretense, sordid desire, hypocrisy, abound. It is to such a World, if at all, that we must appeal.

Shall we then yield? Abase the Queen of the Sciences to sordid uses? Forget the splendid heritage of great men and great thoughts handed down to us through the Ages? Do you expect me to advise that? Is this the road to public comprehension of the worth of mathematics?

No! And a thousand times, No! Leave to those faddists who proclaim each novelty in education to follow this sordid path to quick fame and to future disrepute. We have a surer road. The madnesses of this present hypocritical age, in morals, in finance, in international relations, in education, are knit together, and are bound to swifter failure than could have been hoped a short few years ago. Let our road be the road of honesty. We need no other. This subject which we represent has outlived every form of madness, every sham, every fanaticism, every false goal of human ambition; emperors and dictators, Roman empires holy and unholy, alchemy and astrology, Crusades and revolutions, Kant and Loyola, Berkeley and Voltaire, Platonism and Asceticism, educational theories disciplinary and non-disciplinary, foolish and wise. From the beginnings of Time, through the Golden Ages of Greek dominance, through the Middle Ages, through industrial and social revolution, through educational somersaultings, mathematics has remained measureably intact. It will remain so. Why?

The real merits of mathematics, its real meaning to the World, the World's need of it, remain, and will remain forevermore. To these let us adhere. No propaganda of dazzling novelty appeals to me by comparison with these solid qualities that have brought mathematics down to us undamaged through the whole history of our race. Let us speak only those mild truths whose sureness has been so effective throughout all time. Let me make no oration filled with glowing phrases, no propaganda, no hypocritical appeal to sordid views of get-wise-quick education. Let me rather recount to you those sterling

qualities that have endured, to which this modern world stands strangely indifferent, for which the educational racketeers have only contempt, because there is no novelty, no jazz, no chance to stand and cheer.

2. *What is mathematics?*—The tendency to yield to the clamor for shallow popularity has indeed affected us. There is no dearth of men ready and able to speak to you, as I shall not, of novelties and revisions, of cure-alls and popularizers; men too who can more effectively join phrase to glittering phrase to make smooth sentences. We have indeed a host of books on practical mathematics, on mathematics made easy, on mathematics for engineers, for chemists, for economists, on modern mathematics, on essentials (whatever that may be), on mathematics for modern life, on mathematics of today, on mathematics in general (without any particulars), and we are in a fair way to emulate the horseless carriage in courses allegedly on mathematics with the mathematics left out.

I speak of mathematics, unqualified. I believe firmly that the World needs it. That our life and our civilization cannot go on without it. That it needs no propaganda, nor any oratory. What is then mathematics? What primary qualities have given it such lasting strength throughout all ages?

Formal definitions of mathematics have been attempted rather often. Many definitions, sober and frivolous, are widely known and quoted. To Hamilton, algebra was the science of pure time, and geometry the science of pure form. To others, the logical qualities predominate; these say that mathematics is the science which draws necessary conclusions; but they forget that this is done, more or less imperfectly, in every realm of human knowledge, and that the processes of mathematics are themselves only relatively secure. To Bertrand Russell, philosopher and wit and newspaper writer, mathematics is the science in which we do not know what we are talking about, and do not care whether what we say is true or false. What sense there is in that is only another classification of mathematics as a branch of logic. More recently, in his book entitled *Philosophy* (Norton, New York, 1927), Russell holds that mathematics is an empirical science. Such variation seems to show that, as a mathematician, Russell is a very good newspaper man.

The quality that persists in mathematics is not logic. The meaning of mathematics to the world is not its axiomatic scheme of deduction.

We have, in fact, only been forced to that by the awkwardness of persistent contradictions that have arisen throughout mathematical history, as they are arising in modern physics today. If this be doubted by any of you, I recommend that you study the real origins of geometric knowledge; that you study the furious controversies that surrounded the discovery and introduction of the Calculus, the first forms of which (and some of the present teaching) are decidedly illogical; that you study the equally furious controversies of present-day logicians and intuitionalists in mathematical foundations. Such controversies concern the logician, the philosopher, the scholastic recluse.

Mathematics—the mathematics needed sorely by the World—is none of these. This is not what mathematics means—or can mean—to the World. Such philosophical controversies bear to the mathematics of the World something of the relation that comparative theology bears to religion. What religion means to the World is not theology. What mathematics means to the World is not a branch of logic.

I do not decry theology. I do not decry logic: some of you will know that I myself have dabbled in it; I may as well confess that it is my purpose to speak, at the Atlantic City meeting of the American Association for the Advancement of Science, in my retiring address as Chairman of Section A, on the present controversies regarding the foundations of mathematics. But this is certainly not what mathematics means to the World. The Law of the Excluded Middle would be as far removed from the meaning of mathematics to the World as would the disputations regarding the number of angels that could be accommodated on the point of a needle be from the *religion* of the World.

Mathematics has persisted, and will persist, for qualities of another type. It arose, indeed, from the absolute necessities of the World to deal with geometric forms, such as the land areas in the Nile Valley and the paths of the heavenly bodies, and with the need for counting and for computing. These needs were early needs, and they were found by the World before they were investigated by the logicians. Passing over many centuries, the needs which gave rise to the calculus were again not the mental gymnastics of philosophers and logicians; in fact, such mental gymnastics had been indulged in by the Sophists on precisely the concepts that led to the calculus, but with no better result than those sophistical and sophisticated paradoxes, such as that

of Achilles and the Tortoise, which has in it precisely the same basis for mental confusion that faced the early creators of the calculus, precisely the same (I may add) which makes some modern presentations of the theory of incommensurables quite unsound and illogical. The calculus was the result of the work of Newton, and before him Galileo, Kepler, and Fermat, on moving bodies and on speed and acceleration. The forms they set up were of great service to the World for two centuries before a logical foundation was worked out for them.

Such is the mathematics of the World. We meet, as our fathers met, situations which demand solution. Wherever we turn, these situations are at first vague and formless, as in modern economics; but they gradually take form and are measured first crudely, as were even geometric magnitudes, and then ever more effectively. Modern physics is now measuring all of its quantities; modern chemistry is dealing even with rates of change in measured form; even modern economics is turning to measured forms. With sometimes groping step, as even in early geometry, men learn to estimate, to measure, to find ways of connecting some measures with some others, to set up rules, and finally to write down formulas that express relations and that afford means of human control of quantitative relations. When that happens, a mathematical science is born. Generations thereafter, philosophers, logicians, sophisticated recluses will quarrel about the foundations of it all. Meanwhile the World moves on, controlling better the conditions that it faces because it can at least imperfectly measure them, see the relations between them, predict and control their quantitative amounts. Such, in my opinion, is what mathematics means to the World. This also, rather than the disputations of logicians, is the great need of the World. This is the great quality which has maintained mathematics in its essential forms throughout human history; and will maintain it. This is the mathematics that we can, and should, and must, interpret to the World.

Anything less sweeping than this concept is unworthy of mathematics. To subject things which face us to analysis and to measurement. To find out the relations that exist. To learn by any and by all means—logic—symbols—words—thought—experiment—inspiration—by *all* means, how to control and to predict the measures of the things that confront us in our lives.

Petty definitions sink by contrast to insignificance. To think of algebra merely as the use of symbols for quantities is to distort and

abort and belittle it. Leave that, if you will, to those of our educational brethren who deal by preference in vague concepts without definition or measure; who seek to set up a definition of algebra and of mathematics that is easy to discredit; whose plausible arguments destroy the straw-man of their own erection. True, the World uses few symbols. True, search through, as some have done, our newspapers, our magazines, our books, even those on science, and you will find few *x's* and *y's* and *z's*. Perhaps an *n* now and then, or an *r*, or a *t;* but those do not count to the zealot whose sole aim is to destroy: to be *algebra*, and to count *one* on the scoreboard of quick education, there must be an *x;* if there is no *x*, score zero. Algebra is and always has been and always will be, any (intelligent) discussion of direct relations between measures of quantities. To discover *algebra* in the world, in life, in civilization, the test (for fair-minded men) is to look for (intelligent) discussion of relations between measures of quantities. Do these occur? Look! Look! What field of human activity is free from such measured quantities? What intelligent men in these fields are there who ignore or are ignorant of relationships between them? Such is the true measure of what mathematics means to the World. Such is the interpretation we may and should and must give to it. Such is the World's great need of mathematics throughout all time.

Have we done this? Do our students know how to control and to predict the quantitative situations that confront them? Have we shown them what mathematics means to the World? I shall through limit of time mention in some detail only one, curiously from the *Saturday Evening Post;* but it affects the lives of quite a hundred million fellow countrymen of yours. Just now, let me conclude, however, an attempt to describe mathematical subjects in terms that are justly broad, rather than in terms preferred by those who would destroy it.

Geometry is to many minds again a branch of logic. Their courses have left this impression. As a school subject, it is attacked always on the assumption that it is nothing more than a branch of logic. The facile argument is that logic may be learned in other subjects just as well, and that the logic of the street is not the logic of the class in geometry, but rather that loose, superficial argumentation of debating or of the social sciences. True. The logical foundations of geometry will always remain a monument to man's mental power. I will not defame it. But it means more than that to the World. The control of form, as in a modern bridge, in a modern skyscraper, in the beautiful architectural

dreams of ancient and of modern days, in the vast scheme of heavenly bodies, in the flow of fluids, of late so magnified in importance through streamlining of airplanes and automobiles, of myriad forms of things useful and artistic. Such is what geometry means to the World: form, and the control of form, and all that depends on it.

By contrast, the logical side of geometry is not significant to the World. Does it appear to you that I have degraded geometry? Are not the things that I have mentioned vital, and beautiful, and dignified? It is not that I myself dislike logic; I am trying to say without prejudice or emotion what mathematics means to the World. To it, geometry means the things that I have just mentioned. We should, we must, give that interpretation to it. We fail to reach the World, we fail as fail theologians when they speak theology to a world that wants and needs religion. That people do not appreciate this phase of geometry as a result of our courses, is apparent on every hand. A picture that remains with me is of a young woman (who had long since "finished" geometry) who was walking with my sister in Chicago under an elevated structure that was being riveted together. Emerging from the nerve-racking noises, the only thought that she could summon was to say: "Blanche, I wonder how they ever know when they have got all the rivets in." To her, to many, calculation, structures, bracing, are merely an incantation, an inspirational affair, not a calculation. Another common case is the wonder shown by many who should know better how in the world artillery can be aimed to hit a target that cannot be seen. To such, geometry probably remains only a branch of formal logic.

*3. Aims in education. Aims in mathematics.*—I shall have time to speak only in passing of the aims of education, and of the aims of mathematical teaching. Elsewhere, I have commented at some length on these subjects, and I shall weary some to whom my views are already familiar if I repeat them here.

Let me rehearse briefly not my own views, but some of the principles of education that seem to be accepted by most of those who have given thought to them. We can no longer hold, I suppose, that traditional form of discipline which consists of work in and for itself, as a proper training for the entire human mind. That work must be done is admitted by all; softening of our education to meet weak mentalities is advocated rarely. But we in mathematics have so much that requires work which is also purposeful and justifiable on other

grounds, that we need care little about any further controversies on discipline; we may, and we should, choose topics not solely because they require work, but quite on other grounds. There will be work enough.

Education for life, appreciation of our civilization, ability to meet the life-situations that confront us, are some of the valid reasons for the teaching of school subjects. Mathematics does meet such needs, unless we restrict and deny them. Even if we do, as has been the case in geometry, the real needs will be supplied in spite of us, as has been amply proved in such subjects as descriptive geometry, graphical statics, and other topics now divorced from mathematical departments.

For the learning process, a prime requisite is that the learner be interested in the subject-matter. This prime requisite is admitted, I think, by all. It should not be forced to the extremes which we have often seen: to induce a student by superficialities, by pictures, by amusements, to apply himself to some subject that is not fundamentally dependent upon the devices is an abortion of this principle of interest. In a certain high school recently, in a class in biology, I saw magnificent doll-like creatures made by the students to induce them to interest themselves in the underlying sections of muscles, blood vessels, and bones, but the dresses on the lady-dolls, and the sweaters with crimson H's or blue Y's, got most of their attention. Interest, to be genuine, must be stimulated by direct comprehension of the values of the thing studied.

*4. What mathematics may mean.*—In terms of the aims and principles which I have just stated, what may mathematics mean to the World? What may it mean to the students before us?

There are, of course, those vast and intricate fields of technical application of mathematics, such as the applications to physics, to engineering, to insurance, to architecture. These you may suppose I meant to discuss at length. I do not, both because they are relatively well known, and because I think that we have usually gone about as far with such things as the students' present capabilities of comprehension justify. While I would not slight them, I wish to mention less usual, and less intricate things.

Of the less usual, I wish to refer in passing to a Colloquium held by the American Chemical Society at Indianapolis last year. The subject of it was precisely the needs of mathematics in chemistry, and

the presentation was by chemists, and for chemists. The report of that meeting can be secured readily, and I think it will interest you, particularly if you do not know how much mathematics means to the world of chemistry.

For engineering, a similar colloquium was held at Minneapolis last September; the reports of it are appearing in the journal called *Engineering Education,* the official journal of the Society for the Promotion of Engineering Education.

In economics, it may not be known to you that there has been formed a Society of Mathematical Economics. *A Treatise on Economics* from the mathematical side by G. C. Evans, of the Rice Institute, himself a mathematician of world-wide repute, may be a revelation to you, and to many students. A committee of the Social Science Research Council is now very busily engaged in the preparation of a Report on the mathematics needed for majors in social science; I have a preliminary draft of it in my hand. It will surprise you, I think, to know that they advocate the requirement of mathematics up to and including the calculus. Curiously enough, when this report reached me, I had just been debating with other men in mathematics whether such a requirement could be made and enforced for *mathematics* majors who intend to teach mathematics! It may indeed surprise you to know what mathematics means to the world of social science.

But there are modest questions, also. The planning of a home. The reading of house plans. Drawing of statistics of the home, say of the growth in weight and in height of a child, in a time-scale. Variation of diet, mixtures. Some simple bracing of a shelf, or of a floor. The estimation of volumes of canned goods, without fallacious reasonings. (If a can is twice as tall, it will hold twice as much; if a can is twice as wide in diameter, it will hold twice [?] as much!) These all score zero on the zealot's list, when he is seeking in what ways mathematics enters into life. They loom large in the eyes of simple people whose lives cluster around just such activities. They are, or are not, recognized as mathematical, according to the views that we have taught these people.

Such simple things, such ordinary activities, escape notice. It would require a long time to catalog them all for you here; but I commend these homely uses of mathematics to your especial notice, for it is through them that the humbler walks of life know mathematical quantities and relationships.

One such, still simple, but involved enough to have deceived thousands of people, nay, millions, is the one illustration which I wish to outline in some detail. I have said that I was led to it through an article in the *Saturday Evening Post:* an article by Joseph P. Day in the issue for February 6, 1932, entitled More and Better Mortgages. There are no *x's* and *y's* in it. It would score zero on the zealot's chart. But there are some sober thoughts on quantities and on relations between quantities. One heading is "Buying a Home with Rent Money." Another is "Second-Mortgage Money." Under this he says that, in spite of laws against usury, actually means have been devised to charge as high as fourteen per cent, seventeen per cent, or more, against the little home-owner. He illustrates by quoting a conversation that he had with such an "expert." " 'What do you charge for second mortgage money?' I asked this man. 'Six per cent,' he said, and grinned. 'Actually, how much?' I insisted. 'Twenty-eight per cent is deducted for costs in advance,' he said, 'That is, if a man sells us a second mortgage for $5000, we give him $3600 and then collect interest for the loan at the rate of 6 per cent on $5000, which principal he must repay at the expiration date.' "

Will you figure the real rate of interest paid in such a transaction? Can the little houseowner figure it? Suppose the loan is for three years, with legal interest at 6 per cent, what is the rate actually charged?

Today millions in this country, many unemployed, are paying on such contracts. For homes, for automobiles bought on installments, or "re-financed," for radios, for loans of actual money. The one thing they need to know is that compound interest works like geometric progression. Do they realize this? Doubtless every one of these millions had had geometric progression. Do they recognize it? Can they control the problems of their lives? Do you, for that matter, know the ruinous rates which are being paid by those women and men who have been told that their mathematical problems would be done for them by "experts"? Well, they have been *done* by "experts"!

This then, is the one detailed statement of a problem of relationships between quantities which I have selected to lay before you. It concerns millions of our fellows. Many of them desperate. It concerns our legislatures, whose laws against usury are flouted. It concerns teachers of mathematics, who should prepare students to understand and to control those quantitative situations which they are likely to meet in the struggle of life. Can we neglect it?

This one situation, and the allied questions of bond issues, insurance, pensions, and so on, are quantities measurable and calculable, and, for the most part by means well within the range of the mathematics of the high school. That these are distinctly problems of life, that training to meet and to control them, does constitute "education for life" must be admitted by all except the most confirmed enemies of mathematics. Maybe they do not know, maybe the millions affected do not know, maybe their teachers of algebra did not know, that this again is a part of *what mathematics means to the World.*

*5. Why the meaning of mathematics is not taught.*—The fact that the situation which I have just outlined can exist, the fact that most of those who have studied mathematics in high schools do not even conceive that it may be within their powers, raises in my mind a typical question. Why is it that the meaning of mathematics is not brought out, and brought home to our students, in this and in many other connections.

One answer which I have publicly discussed otherwise is the tremendous tendency toward formalism in mathematical teaching. "Millions for factoring, but not one cent for compound interest" might be a typical motto. Factoring? Yes, tons of it. And withal with so much formalism, that the very teachers whom I have seen carry factoring to the absurd length of factoring the difference of two $n$th powers, did not themselves know, when they reached geometric progression, that the sum of a geometric progression is precisely the quotient of this difference of two $n$th powers by its simplest factor. Such is formalism gone mad. Formalism driven to the length of a repetition because the essential truth covered by the formalism was not apparent even to the teacher of it. And hence another formalism, to the exclusion of any mention of compound interest.

I shall not outline here, as I have done elsewhere, the variety of causes that tend toward formalism in our schools. One only seems to me to be particularly germane here. It is the question of the training of our teachers, a subject just now under intensive study by the Federal Bureau of Education, and by the International Commission on the Teaching of Mathematics, of which Dr. David Eugene Smith is the International Chairman. These matters are strongly before me, since I have the honor to be the Chairman of the American Committee of the International Commission, the other members being Prof. W. D. Reeve, of Teachers College, Columbia, Prof. Eva M. Luse, of Iowa

Teachers College, Dr. Ben Frazier, of the Bureau of Education, and Mr. Ben Sueltz, of the State Normal School at Cortland, New York. When the Federal Survey has been completed, and has been digested, as Professor Reeve and some of his students are now doing for my committee, we shall know in detail about the training of teachers in the whole country. Already, however, we know that it is worse than your most pessimistic fears, in instances in which statistics do exist. Whole states exist—in the North—in which one-third of the high school teachers of mathematics who have a college degree, took not one hour of mathematical work in college.

Shall we expect such persons, demonstrably lacking any interest in mathematics, themselves to inspire their students to be interested? Shall we expect them to show *what mathematics means to the World?* Shall we expect through them to escape from improper formalisms?

*6. This hypocritical world.*—The world in which we live today is mad with the craze for speed, mad with the craze for sudden wealth, deaf to the call of the spirit, derisive of all that is cultural, rotten with hypocrisy and cant. From prohibition on the one hand to what E. T. Bell has called *Racketeering in Science;* from "experts" in finance who can make twenty per cent look like six, to tawdry religions that are infamously sacrilegious, from "patriotic" societies whose leaders are aroused only by a prospect that the government is to be asked to keep the poor from starving to educational propagandists who serve only their own fame, the stench of hypocrisy arises on all hands.

To such a world we must appeal. Appeal; yes, but for reform, for honest dealing, for plain truth, against such hypocrisies as those we see nearest us. One such is the teaching of mathematics by those who themselves despise it. By thousands, there stand before our children at this time, teachers pretending to arouse interest who are themselves not interested. Such things must cease, or mathematics itself is and should be doomed. Teachers must be trained both to know the aims of education, and the consequent aims of mathematics; and too, to know the subject matter of mathematics itself. They are false leaders of education who say that the knowledge of the subject alone is sufficient, and that no training in imparting it is needed. They are also false leaders who claim, foolishly, that training in subject matter is unnecessary.

The economic world is staggering, and may fall, because the people have come to know that the financial leaders had betrayed them, be-

cause faith that was very strong in the abilities of our men of business affairs is shattered, because men said to be public benefactors were found serving only self and greed. How near we are to complete collapse of the economic structure through such causes you know only too well.

The people have shown equal faith in educational leaders. No appeals for funds for education in the United States have fallen on deaf ears. I tremble when I consider what may occur when or if the people learn that educational leadership has not been wise, not been unselfish; learn the things that I have been telling you; learn what you know of educational faddism and of educational racketeering. You know, as I know, that the sound, eternal truths, of values in education have been distorted and perverted. You know, as I know, that mathematics has been attacked, that other solid subjects have been assailed, until one may wonder, as the superintendent of schools of one of our largest states has just asked in a published article: "What is the purpose of secondary education?" Such assaults as those that have been made for selfish ends, and for notoriety, by asserted educational leaders, weaken our whole structure, and lay us liable to just such a debacle as that which is now actual in the business world.

Educational requirements set for selfish purposes, on the one hand contain no element of training in teaching nor in the aims of education, on the other hand contain little or no training in the appreciation and knowledge of the subject-matter itself. Both these are selfish, based only on greed and on hypocrisy. The situation in mathematical teaching is not sound. It is leading to the teaching of improper formalisms that are not mathematics, and to a false view of what mathematics means to the world. It is maintained by special interests and for selfish purposes, in school systems, in departments of mathematics in colleges and universities where the aims of mathematical teaching are not instilled, by departments of education in which the knowledge of subject-matter is decried, by educational propagandists for this or that novelty in education who seek to discredit mathematics, and by a general apathy in the public mind which is absorbed by the mad scramble for what the world calls success.

Of late I visited again, alone, a very famous university where I spent many happy hours of youth. It happened that I saw that day no familiar face, no friend; and every aspect of the place was changed by

great new buildings, some of them frank monuments to Mammon. I entered through a gate on which I read this noble saying:

OPEN YE THE GATES, THAT THE RIGHTEOUS NATION
WHICH KEEPETH THE TRUTH MAY ENTER IN

Which is this nation? Ours? I went, unanswered. To you, I say that unless this nation turn from its ways, unless there arise through these sad times remorse, reform, repentance; unless our leaders in affairs, in politics, in religion, and in education, turn their backs on pretense, on greed, on hypocrisies, the Gates of Progress will be shut against us.

Forget whatever of my statements have been wrong, or prejudiced, or exaggerated. This is no time for that. No time for oratory or for propaganda. No time but for the naked truth. Remember only that I have tried to recall to your minds the eternal values, in mathematics, in our whole lives, in economics, in affairs, in religion, in education. May you hold to them. It is in the power of you—teachers—to sway our whole population, to re-make this sorry world; these children now under your guidance *will be* the nation of the future. Shall it be a *righteous* nation? What will you instill into their minds and hearts? Will you teach them eternal truths and righteousness? So like a call to arms, I read again to you those same noble words, with a new emphasis: it is for you to determine that truth shall prevail, it is for you to fight greed and to stamp out hypocrisy; and you, as a plea for duty, let me adjure:

Open YE the gates, that the RIGHTEOUS nation
which keepeth the TRUTH may enter in!

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## Notice to May Subscribers!

If your subscription to The Mathematics Teacher expires with the May number, it will be so stated on the outside wrapper of that issue. Please send in your annual dues of $2 at once so that your files may be kept complete.

The Editor

# Curriculum Adjustments*

*By* W. C. MEYERS, *Assistant Principal, McKinley Manual Training School, Washington, D.C.*

BEFORE CONSIDERING the subject "curriculum adjustments" as they apply to slow moving pupils, it will perhaps be worth while briefly to call attention to some historical facts regarding the growth of mathematics, in the high school and college courses. The teachers of mathematics, like many school officials, are inclined to assume that the mathematical content we are teaching today is the same as that taught by our forefathers. This is a false assumption: quite the contrary is true. Our present mathematical curriculum has been with us hardly long enough to have a tradition, certainly not long enough to have acquired a sacredness which must be worshipped.

In 1800 Harvard required no mathematics for entrance, Yale and Columbia required vulgar arithmetic including the Rule of Three for matriculation. The rapid growth during the next hundred years is clearly shown in a study of the entrance requirements of 434 universities, made in 1896-97. At that time it was found that 412 required algebra for admission, some of them requiring to quadratics, some through quadratics, while the great majority did not specify the amount. The majority then required plane geometry, 92 of them required solid geometry, 8 required spherical geometry, 2 required conic sections, and 4 trigonometry—a tremendous crowding down to pre-college age, what had been previously given in college or not at all.

The content of mathematics in our high school curricula reached the maximum at this period. It is certainly true that since 1900 mathematics as a prerequisite to college entrance has declined and with this decline there has been a like decline in the graduation requirements of mathematics in the secondary schools.

During the years from 1918 to 1924, in 50 leading colleges and universities, there was a reduction of 20.3 per cent in the college entrance requirements for mathematics. During the same period of time, there was a reduction of 75.1 per cent, in these same institutions, in the requirements of mathematics for the A.B. degree, clearly showing that, at the present time, the pendulum is swinging away

* Read before the National Council of Teachers of Mathematics at its annual meeting in Washington, D.C., on February 19, 1932.

from mathematics as a required subject both in high school and in college courses. This brief outline ought to raise the question as to whether the serious problems confronting us are not due to past excessive zeal in crowding down more and more mathematics upon the unprotected heads of our boys and girls—more and more of whom are ranging around the lower end of the ability scale. Dr. Betz in THE MATHEMATICS TEACHER of February, 1930, says:

> It is necessary to remember that the high school is enrolling an ever-increasing number of boys and girls from the less favored classes of society. The genius, the normal, the halt, and the blind come to our schools on equal terms. This has involved a lowering of intelligence, if by intelligence we mean the ability to meet traditional standards within the customary time limit of one year. According to a safe estimate, at least 200,000 now studying elementary algebra cannot master the prescribed course within the allotted year's time.

One of the first attempts to handle the situation created by this intelligence decline was that of elimination. We cut out this subject, and chopped off the more difficult parts of that. Then turned around and slipped in a bit of trigonometry, and intuitive geometry with a dash of analytics on the unsuspecting youngster. But in spite of all efforts, mortality in mathematics remained about the same. Well, something had to be done—or mathematics would lose caste as a high school subject. With the coming of the intelligence quotient, many gladly grasped at the ability grouping idea as an effective remedy that would successfully combat this high mortality. Somewhat arbitrarily we have divided the intelligence scale into three divisions and have had published textbooks with three rather well defined courses, the differences in which consist almost entirely in quantity of work required (this is especially true of algebra).

In our own school—the McKinley High School of this city—we call these groups 1, 2, and 3—group 3 being the slow-moving one. Taking this school as a typical high school (and I feel that it is typical in at least two respects, namely, in its enormous increase in enrollment, and in that the student body occupies a mid-position in intelligence) taking it then let me briefly sketch some of the results of the ability grouping experiment as worked out in it. The grouping idea was introduced with the avowed purpose of doing two things:

1. To eliminate the retardation of the fast moving pupil.
2. To reduce the high rate of mortality (mathematics being one of the chief causes).

In regard to the first, while I have an opinion, I have no facts to substantiate it, so I will hazard no judgment as to whether the purpose has been or is being accomplished.

As to the second, that is, the reduction of the high rate of mortality, it has been materially reduced—but whether this reduction is due to the merits of the system, or to the lowering of teacher standards, or other factors involved, I am unable to say.

But granting that it has loosed the fetters from the fast moving pupil so that he can progress at a normal rate of speed, and further granting that the system is responsible for this reduction in the total mortality rate—I submit the following figures in an attempt to show that we still have a problem that demands further attention.

During the past three years our average enrollment in elementary algebra and plane geometry has been in the neighborhood of 1000. And, each year, approximately 40 per cent of these pupils received a group 3 mark, and of these slow moving pupils 34 per cent failed. That is, of two-fifths of our enrollment, we make demands that approximately one-third cannot meet. Surely, if this is a typical situation, it should constitute a challenge to mathematical educators.

Arithmetic is frequently mentioned as a way to meet this challenge. L. E. Mensenkamp, in the January, 1929, MATHEMATICS TEACHER suggests it as a solution for the problem, saying that the transfer of a slow moving pupil to arithmetic would decrease the per cent of failures. No doubt this would be true so far as algebra is concerned, but it would seem merely to transfer the high percentage of failure from algebra to arithmetic. Statistics gathered in our school indicate that this is so, for since the introduction of the grouping system in the school, the arithmetic classes have increased steadily from year to year and along with the *increase* in numbers there has been a gradual increase in the percentage of failures, until at the present time arithmetic has the distinction of having the highest rate of mortality in the school—more even than the deadly Latin and geometry. In some recently published statistics on the percentage of failure in the various subjects in a school system where there are in the neighborhood of 12,000 pupils in the various high schools, it was shown that the percentage of failure in arithmetic was the highest of the list, about double that of algebra and geometry. Thus it would seem that the transfer from algebra to arithmetic is merely a juggling trick and not a solution of the problem. Furthermore, psychologically

the above results would seem to be the logical outcome, since obviously the previous arithmetical experiences of this group of pupils have for them little or no meaning.

Dr. Judd says in the *Third Year Book*, "It is certain that the schools of our day do not know how to teach arithmetic—(The increasing failure each year up to the fifth as shown both by the Cleveland survey and the California curriculum study, clearly demonstrate this)."

The high percentage of failure, when the pupil comes to arithmetic in high school, would seem to indicate either that Dr. Judd's conclusion from the findings of the Cleveland and California surveys hold true for the high schools as well as the grade schools, or that through poor preparation or deficient mentality pupils are unable to follow arithmetic processes. Hence repetition has little or no value.

Now, in thinking of this group, whether we conceive it to be composed of the mentally deficient, or the mentally deficient and triflers, is immaterial. The fact remains that when they reach the high school, algebraic and geometric symbols have little significance for them. In a study of over 3000 pupils in the Hartford schools, noted on page 12 of the Government Document No. 34, the statement is made that the mentally backward leave high school in increasing numbers during successive years, while the triflers remain. If this be quite generally true, then the tinkering with, modifying of, and patching up of our curricula to meet the needs of these ability extremes has not succeeded but has merely driven out the mentally deficient and has made more confirmed loafers of the triflers—to say nothing of what it has done for the brighter pupils among whom we must look for our future Euclids, Newtons, and Einsteins. According to Professor Thorndike's conclusions from his investigations, we are giving them mastery of nothing. He further gives it as his opinion that pupils whose I.Q. quotients are less than 110 are unable to understand the symbolism, generalizations, and proofs of algebra. If this dictum be accepted—and our four year experiment inclines me to accept it—then, with at least one-half of our high school pupils we have a task in teaching them mathematics beside which the ball-rolling stunt of Sisyphus was child's play.

Dr. Hedrick, in the *Third Yearbook* raises the question as to whether after all the real value of mathematics does not lie in the process. He lists three fundamental specimen processes which, if ac-

quired, would contribute to real life value. As outlined, they are:

1. Precise statement
2. Generalization
3. Necessary and sufficient conditions

just the mental processes and abilities so necessary in life and yet denied by Professor Thorndike to at least 55 per cent of the pupils enrolled in our school. Now—whether the dividing line between those who can grasp the symbolism of mathematics and those who cannot, be at 40, 50, 60, or any other number of per cent—it is certainly a fact that in the high schools there is a considerable number whose life's experiences have not made even "word symbols" meaningful, much less the more complex ones of algebra and geometry.

If Professor Thorndike's conclusion that an I.Q. of 110 is the dividing line between those groups, then would it not be the part of wisdom to recognize two groups, those who can and those who cannot understand the concepts and symbolisms of algebra, proceed to demand a real mastery of something from those able to follow intelligently these processes, and detach the others from the mathematical train and build a curriculum around their interests, designed to develop the simplest ideas of form, precision, arrangement, and orderliness. Such a course should make no attempt at subject matter transfer, but should be entirely functional. (Perhaps it should be built around the club, hobby, or vocation idea, which at the present time seems to be all the rage. Dr. Jessen, specialist in secondary education of the Department of Education, says that in a study of high schools made in Ohio recently, the one thing that appeared to the department rather remarkable was the absence of any extra-curricular activities growing out of the subject of mathematics.)

In my judgment, two groups rather than three, will be the precipitate left when the experiment is finished and the results tabulated. Dr. Billett, of the Department of Education, specialist in school administration, after carrying on a series of experiments to determine the validity of a threefold system says:

"There is in general but one recommendation to be made, namely, that slow moving pupils should be segregated for instructional purposes, while the average and fast moving pupils should be organized into heterogeneous groups."

His study shows that the slow moving are slightly benefited by segregation while the fast moving ones are hindered.

He would include in the highest group the average moving group

as well. The slight improvement noted in the slow group was achieved by means of the present patched up curriculum wherein the difference is quantitative only. Now I advocate that we make an effort to add to this small gain a much larger possible gain by making the course different qualitatively as well as quantitatively.

I am looking forward to the day when some mathematical Moses will appear with the courage to scrap the old traditional book approach to mathematics for the mentally unqualified, and to give us what Dr. Judd so ably pleads for in the April number of THE MATHEMATICS TEACHER under the caption of "Informational versus Computational Mathematics." His closing paragraph is worth quoting.

> When one recognizes the historical fact that a long procession of textbooks in mathematics has followed antiquated models so faithfully that it is almost impossible to tell them apart, one realizes that a revolution may be necessary to disturb the complacent slumbers of the teachers and textbook makers in this field. Perhaps American schools will have to be deprived for a time of the full benefit of the kind of mathematics now current in order that mathematicians may be persuaded to mend their ways.

And where can a better place be found to begin mending our ways than on this slow moving group; and what body of educators is so well qualified as this distinguished National Council of Teachers of Mathematics? To take up his challenge and proceed in a scientific manner to determine whether grouping by means of the intelligence quotient is desirable in mathematics, and if so, to work out such a curriculum as will contribute to the development of personality in this mental type of pupil.

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## News Note

THE MATHEMATICS SECTION of the twelfth Ohio Educational Conference was held at the College of Education at Ohio State University on Friday April 8 at 2 P.M. Professor R. L. Morton presided.

The following program was given:

Some Worthy Special Objectives for Secondary Mathematics. Paul C. Recker, Toledo public schools.

Protecting the Social Investment in Arithmetic by a Margin of Safety, Carmille Holley, Miami University.

Mathematics as an Interpreter of the Social and Economic Environment, W. S. Schlauch, New York University.

# Improving America's Mathematics*

*By* HARRY C. BARBER, *Exeter, New Hampshire*

I BEGIN WITH a protest against a point of view. It is held in certain quarters that the way to improve America's mathematics is to go to the man in the street and to learn from him what mathematics he is using today, and then to teach that mathematics in school. The mathematics used by the ordinary citizen in the pursuit of his ordinary tasks is the mathematics to teach, and it is all the mathematics that ought to be taught, to the ordinary citizen. That sounds plausible does it not?

It may be observed, however, that other bodies of professional people do not proceed in exactly that way. Physicians, for example, do not ask the man in the street what pills he is taking today and what inoculations and operations he believes in, and then teach the medical students or even the general public only those pills and those operations. When physicians adopt that method they will surely begin to retrogress.

We ourselves are a body of trained professional people and we ought to be able to tell the public what mathematics it needs to know and how it should think about mathematics in this scientific age. Never has mathematics been so essential to an understanding of what the world is doing. One hardly advances, in his reading, beyond the tabloid newspaper stage before he arrives at scientific ideas which call for some knowledge of mathematics. To be baffled by a simple formula, for instance, is to be cut off from an ever increasing sector of popular scientific literature; it is to be cut off from a method of talking and of thinking with which everyone in this age ought to be familiar.

We protest, then, the static theory that would limit the mathematics of the public to that about which the man in the street is already articulate. Of course we need to look around, as does everyone else, in order to make sure that we perpetuate no cumbersome and antiquated process. But for everything useless we omit we should substitute something indispensable, and we cannot depend upon the man in the street to tell us what it is. We must tell him. More than

* Read at the annual meeting of the National Council of Teachers of Mathematics at Washington, D.C., on Saturday, February 20, 1932.

that, we must convince him. We ought to give him mathematical ideas to think about, ideas which make his life more effective and more interesting. We ought to give him methods of thought, habits of thought, which develop a taste for rather nice thinking and a distaste for slovenly thinking; habits of sifting evidence, of reserving judgment until the evidence is in, of drawing sound conclusions, and without prejudice checking those conclusions against the facts.

### *Some Implications*

If we postulate the duty of improving America's mathematics we are faced with various implications. We must know how to make our mathematics fit the present age. We must recognize the elementary mathematical ideas which contribute to man's understanding of his environment. We must be able to interpret for the secondary school level the contributions which mathematics is now rendering to the other sciences. We must have well-founded convictions as to what the study of mathematics can do and cannot do for mental habits.

Are we ready to assume these responsibilities? Well! I think we are getting ready. We are studying the problem as never before. The literature of the subject is increasing, summer schools are giving opportunity for study, the National Council of Teachers of Mathematics is beginning to make important contributions.

I am much interested in the committee of the Council, headed by Professor Rankin of Duke University, for investigation of fields of application of elementary mathematics; and also in the committee headed by Professor Schlauch of New York University, for the study of extramural examinations. I often hear discussion of examinations set by the College Entrance Examination Board but I never hear the question asked, "Did this examination direct the candidates' thought along important lines? Did it make secondary mathematics seem worthwhile?" These are surely important questions to ask. In them I hope and expect that Professor Schlauch's committee will arouse widespread interest.

It is obvious of course that we are not now going to make the final statement about improving America's mathematics. All we can hope to do is to direct our thought along some of the lines which can profitably be followed. The moves I here propose are those upon which we can begin at once:

*1. Bring school mathematics into closer relation with other sciences.*

—It should not be difficult for us to take advantage of the fact that *science* is, in our day and generation, a magic word. Educators, well aware of this magic, now advocate the allotment of more school time to the natural sciences, and we ought to foster this movement for more reasons than one. Unfortunately the same educators often advocate less time for mathematics. In that situation we should be very foolish not to ally ourselves as closely as possible with our sister sciences. We might as well bask in the sun which shines on them, or to return to our former figure, we might as well let their magic wand wave over us. We should not have to be very clever in order to show pupil and public what a regal place mathematics holds among the exact sciences.

In secondary schools in England instructors teach mathematics and mechanics side by side to the great benefit of both. President Moore, in his well known presidential address before the American Mathematical Society thirty years ago, an address still available to us in the First Yearbook of the National Council, advocated joint laboratory courses in mathematics and physics. Such courses would be stimulating to teachers and students. Twelfth-grade courses of this sort could use mechanics as an introduction to and a field of application for the calculus. Any of these courses would serve to connect mathematics with science in the minds of all who came in contact with them.

But we need not delay our attack on this problem until such courses arrive in this country. We can now turn to science for applications of the mathematics of every grade from seven through twelve. We who teach can make ourselves very familiar with the field of high school science. Many of us can teach both science and mathematics. In our mathematics classes we can teach, among other things, the elementary, common-sense notions of dealing with the approximate numbers which so frequently occur in science. Familiarity with secondary school science ought to be a part of the irreducible minimum requirement for qualification as a teacher of mathematics. Then too we can be informed in regard to the new developments of science so that we can reply to questions about the fourth dimension, non-euclidean geometry, Mr. Eddington, or Mr. Einstein.

I need not go further. In this presence a mere statement of the problem is sufficient. If you will allow me to change my figure of speech once more—we are all ready and able to ride upon the advancing crest of the wave of science.

*2. Bring algebra and geometry into closer contact with each other.* —It seems to me that it is time to do away with "the theory of proportion." The solution of one type of equation verbally rather than by means of algebraic symbols is easy to explain historically but difficult to justify as a present day mathematical procedure. Why should we go back to the period of verbal algebra when we want to transform a proportion in the geometry class? Once we have set at work to rewrite our theorems so as to do away with verbal algebra, I hope that we shall go the whole distance and replace verbiage by good algebra wherever possible.

I hold it to be axiomatic that if we are to improve America's algebra we must see to it that algebra finds uses in the geometry class. Why should we spend a year or two trying to convince the pupil that alegbra is an important symbolism for thought and then, during the next year in the geometry class, use all manner of circumlocutions as substitutes for algebra? I do not mean merely that we should set up geometric exercises for algebraic solution, valuable as they are, but that we should in our demonstrations prefer reasoning with algebraic symbols to reasoning with sterotyped verbal phrases. I even go so far as to say that I could very well do without ever again hearing that "equals added to equals give equal sums," etc., etc. It would surely make for continuity in mathematical instruction if in the geometry class we added, subtracted, multiplied, divided, and substituted just as in the class in algebra.

One of our experimental divisions this year is completing Algebra A and at the same time doing all possible incidental geometry. The young instructor, a rare good teacher, dropped in the other day to tell me how naturally the class was using algebraic symbolism in its geometry. If we gave the students half a chance they would institute this reform for themselves. If their study of algebra convinced them that algebraic symbolism was a worthwhile aid to thought they would naturally make use of it in geometry provided only that we did not begin by filling their minds with the rather silly verbal phrases which so naturally roll off our tongues.

Trigonometry should have brief mention before we leave this point. A unit of simple trigonometry is now well intrenched in the algebra course. It is, as everyone knows, more closely related to geometry than to algebra and there is every reason for extending into geometry the use of tangents, sines, and cosines. They not only supply important numerical work but they can illuminate the study of triangles

and they can be of use in demonstration. Archimedes in effect used them in his solid geometry argument. They can surely be used to overcome the feeling which must be prevalent among the graduates of our classes in geometry that only triangles containing 30°, 45°, or 60° are capable of solution. If we do not use the trigonometric functions in the mathematics of each grade from the ninth onward, we certainly miss an opportunity for continuity of mathematical growth and so fail to make our mathematics as convincing as it ought to be.

*3. Give the student a better idea of what algebra is.*—When I was a small boy on the verge of the study of the mysterious subject called algebra, I asked an older boy what algebra was. He told me it was "where you use letters." "What for?" I asked, and he replied, "To add, subtract, multiply, and divide." So long as our pupils can formulate such definitions of algebra, America's mathematics will be in need of improvement.

I submit that the way to effect the improvement is to define algebra as a symbolism for thinking about numbers. Then to introduce each symbol and each process at the moment when it can be used in the kind of thinking we are doing. We are engaged in orderly thinking about the number side of life. Algebraic symbols help us in this thinking. Of course, we may eventually want to add, subtract, multiply, and divide letters, but these operations are only incidental to the use of the symbols for some effective purpose. Did you ever stop to think, by the way, that vertical addition and subtraction are not useful in first-year algebra until simultaneous equations are to be solved by combination?

So long as our pupils can define algebra as my boy friend did, we who teach it may be said to have the cart before the horse. If we redefine algebra, as a symbolism for thinking, and if we carry this definition with us day by day into the classroom we shall not only considerably modify our order of attack but we shall considerably modify the reactions of our students, and eventually we shall, I am sure, effect some improvement in America's mathematics.

*4. Stress the unique contributions of mathematics to education.*—Among school subjects which require so much reading and studying of books, so much memorization, it is the glory of mathematics that it gives one the opportunity to proceed under his own power. We ought to take advantage of this difference. Count no day lost when some pupil points out to you that mathematics is different from his other subjects because he can do it for himself. We ought in our classrooms

to change the connotation of the word "learn," to make it mean not learning what some one has laid out to be learned but rather finding out for oneself.

In grades seven and eight students of mathematics ought to be changed from memorizers into discoverers. They ought to make little voyages of exploration. They may find the value of π or the height of the flagpole, or how to solve the equation $\frac{1}{2}x = 7$, or how to estimate the square root of a number, or any of the multitude of mathematical things which they can learn for themselves. At any rate the teacher varies the monotony of recitation with the words, "Let us see what we can find out about that."

I should like to change the connotation of yet another word. The word "drill" ought to mean not only repetition but also reclarification. There are many things in secondary school mathematics which ought to be well understood; there are but few which need to be automatic. Digging into a subject and coming to understand it, is a more effective educational process than merely repeating the result of the investigation. Consequently it is the investigation which ought to be repeated.

The first time a new topic comes up we investigate to see what we can discover. We put our discoveries into practice on a few illustrations. A little later the same topic is needed again. This time one or two of the brighter students will be able to lead the reinvestigation, and by the third time the topic appears all but perhaps the weakest can reason the thing out for themselves.

Do you not agree with me, first, that if mathematics is to yield its most effective contribution to education it must teach habits of investigation, of exploration, of seeking the why as well as the how, what I have called elsewhere the habit of understanding; and second, that the way to develop these habits is to drill on them rather than to drill on mechanical processes alone? I am sure that the teacher who thinks of learning as memorizing and of drill as the repetition of mechanical process only, will have less wholesome effect upon the minds of his pupils than the teacher who thinks of learning as finding out and of drill as reclarification and rediscovery.

### *Conclusion*

My thesis is that we are now a professionally trained body of teachers capable of effecting significant improvement in the manner in which the public uses mathematics and thinks about mathematics.

In bringing about this improvement we can ride upon the crest of the wave of science. We can present simple, convincing evidence of the key position which mathematics occupies among the exact sciences. We can stress the essential unity of secondary mathematics rather than the various categories into which it may be separated. We can exhibit its underlying aims rather than merely its techniques. We can so teach mathematics that it will yield ideas of exploration, of self-discovery; so that it will develop the scientific attitude of mind.

I do not mean that we can talk about such accomplishments when we are gathered together in annual meetings or that we can dream about doing such things in the dim and distant future, but that we can, if we will, accomplish them in our classrooms now.

I believe that the National Council of Teachers of mathematics is already effecting much improvement in America's mathematics. The annual meetings become better and better. So does our journal THE MATHEMATICS TEACHER, under the efficient editorship of Dr. Reeve and Dr. Sanford. If you will go out and build the subscription list up to 10,000 you will see still greater improvement and greater effectiveness.

I should like to have THE MATHEMATICS TEACHER call our attention to the best books and articles in our field throughout the world; to the best European teaching practice; to the advances in thought in our own field and related fields. I should like to have it develop and keep before us some of the fundamentals of what might be called a mathematics teacher's philosophy.

In other words, my final proposal for the improvement of America's mathematics is this. Build up the membership in the Council, that is, the subscription list of THE MATHEMATICS TEACHER. It is a good missionary.

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## National Council Members!

REPRINTS of page 245 at the front of this issue of THE MATHEMATICS TEACHER will be sent on request to all those who will help in securing new members at teachers' meetings, clubs, summer sessions, and the like. Please indicate the number of reprints you can use to advantage.

THE EDITOR

# A Comparative Study of the Teaching of Mathematics in the United States and Germany*

*By* W. D. REEVE, *Teachers College, Columbia University*

*Present tendencies.*—There never was a time when people were more interested in world tendencies. Present tendencies are more or less socialistic in nature and affect the educational situation in every country in the world. These tendencies naturally influence the teaching of mathematics particularly in this country where we have set ourselves up against any aristocracy of learning. One child must have as good a chance as another.

We have more children above fourteen years of age in school in the United States than in all of the other countries of the world put together. Nothing like the American public school system has ever been known before. We have a wonderful opportunity to make or mar our future. At the present rate our pupils will soon be the most generally schooled population in the world, but will they be the best educated?

Mathematics has been adapted to mass education. The natural result is a lowering of scholarship, a leveling down instead of a leveling up, but an increase in influence which on the whole will result in good because more people will have a chance without necessarily jeopardizing our opportunity to discover the pupils of genius.

We no longer have as select a group as we used to have in the secondary school in spite of the fact that our high school population approximates the best fifty per cent of American intelligence. Large numbers of immigrants, or their children, in recent years, ambitious to secure an education and imbued with an earnestness of purpose far ahead of many of our native American children, are clamoring for at least a high school education. The result has been a demand for teachers to teach the 20,000,000 or more pupils in our schools.

*Our insularity of mind.*—There is great danger, however, in our

* Read at the annual meeting of the National Council of Teachers of Mathematics at Washington, D.C. on Friday, February 19, 1932. Reprints of this article may be had from THE MATHEMATICS TEACHER for 10¢ each, postpaid.

insularity of mind. Our American teachers need to learn of the salient features of the teaching of mathematics in other great countries of the world just as much as they need to learn of art, literature, politics, and the like. In fact it is as dangerous to foster nationalism in mathematics as it is to be nationalistic in our political views. If we make a comparative study of our schools and those of Europe, for example, we find that in the fifth school year we begin to fall behind in mathematics, by the time we have reached the end of the eighth year we are one year behind, and at the end of the twelfth year we are two years behind in so far as academic achievement is concerned.

Some one will say that this is what should be expected in a country like ours where the purpose in education is different. For example, the aim in France is to develop mathematicians while here the aim is to develop well educated citizens. Naturally we get what we expect in a democracy of education. We are confronted with at least two possibilities, an eight year elementary school, or a six year elementary school followed by a six year secondary school. All this work is pointed toward a finished product in the secondary school.

*Academic background of teachers.*—The fact that our country is new and that the high school population has increased so fast in the last thirty years has made it impossible for us to demand as high qualification for teachers as we should like to have had.

Another significant factor is the fact that our unprecedented industrial and economic growth has resulted in men leaving the teaching profession to go into other lines of work. The result of this is the domination of women in the classrooms throughout the land. While we may admit the natural superiority of women with children we must not fail to realize that for most of our women teaching is not a permanent profession. What is the mental effect? I think it has led to a lack of interest in the proper preparation and background for teaching mathematics in the schools. As a result we do not have among either men or women teachers enough people of scholarly minds. The mathematical background of many of our teachers is decidedly limited, to say nothing of their deficiencies along other lines.

Most of the teachers of mathematics in the secondary schools of Germany are men most of whom have had academic training in mathematics culminating in the Ph.D. degree. Moreover, most of the teachers of mathematics that I saw at work in Germany can teach physics

equally well. A great many of our secondary teachers of mathematics would have a hard time to follow the ordinary classroom discussions of such teachers.

One would naturally expect to find men teachers in the boys schools (Knaben Schulen) of Germany, but even in the girl's schools (Mädchen Schulen) one often finds men teaching both mathematics and physics. In fact I had a hard time to find women teaching mathematics at all, but the ones I saw were good.

Many people are teaching mathematics today who have not had any mathematics courses of collegiate grade. The situation is even worse than that in some places as the forthcoming report of the National Committee on the Training of Teachers of Mathematics will show. When a superintendent of schools insists on putting a French teacher who has had no mathematical training in charge of high school classes in mathematics because he has no other use for her the situation is becoming intolerable. The National Council should begin to study such practices and make plans to do something about it. We could do it if we had as Professor Slaught once pointed out a sufficient amount of group consciousness, group enthusiasm, group loyalty, and the courage of our convictions.

*Mathematical literature.*—Compare if you will the various types of reports, monographs, magazines and books[1] that are published for mathematics teachers in either England or Germany with those in the United States and you will see that the background necessary to understand the former is far greater than that in our own country. We may occasionally publish an article in THE MATHEMATICS TEACHER that will challenge the best thinking on the part of some of our most able teachers, but we dare not give all of our readers a regular diet of that type. We are now trying to publish whenever possible

[1] The following are good illustrations:

(a) *The Teaching of Geometry in Schools.* A report prepared for the Mathematical Association in Great Britain. G. Bell & Sons, London.

(b) *The Mathematical Gazette.* Official Journal of the Mathematical Association of Great Britain. G. Bell & Sons, London.

(c) *Unterrichtblätter Für Mathematik und Naturwissenschaften.* Organ Des Deutschen Vereins zur Förderung Des Mathematischen U. Naturwissenschaftlichen Unterrichts. Salle, Otto. Fraufurt A.M. und Berlin.

(d) *Zeitschrift Für Mathematischen und Naturwissenschaftlichen Unterricht aller Schulgartungen.* Teubner, B. G. Leipzig and Berlin.

some articles that relate more to subject matter than to methods[2] in order to secure a better balance, but we have not yet succeeded in solving this problem.

The late Professor Young of Dartmouth College[3] at a recent meeting of the Mathematical Association of America in speaking of the function of the Association said, "In the field of secondary education, we might expect the National Council of Teachers of Mathematics to carry on. Unfortunately, from some points of view at any rate, the National Council reflects the point of view of our schools of education rather than that of our departments of mathematics. Their official organ, THE MATHEMATICS TEACHER, seems to me to be almost altogether 'teacher' and hardly at all 'mathematics.' So that here again any real mathematical interests seem to be the function of the Association." That the Mathematical Association is not functioning perfectly in this respect is evidenced by the fact that a little later in his discussion he said, "*The American Mathematical Monthly* through its articles, discussions, and problems provides an additional and effective method of stimulation. The evolution of the *Monthly* from a small struggling pamphlet to its present size and standing forms an interesting chapter in the development of mathematics in this country. We can all rejoice in its progress and its increased influence. As we look back over this evolution, however, we note a marked shift in its appeal. The *Monthly* at present is addressed primarily to the college teacher; it used to be of interest to the college student, as well. Nowadays, it has little to offer the undergraduate, except possibly the senior who is majoring in mathematics. Its problem department is in general beyond the ability and training of the undergraduate in his earlier years."

Just so. What Professor Young said THE MATHEMATICS TEACHER has failed to do for secondary teachers of ability, he frankly admitted the *Monthly* also has failed to do. And I might add that rarely if ever does the *Monthly* do anything for the pedagogical interests of its readers. I would like to suggest to the members of the Mathematics Association that they turn over any worth-while articles dealing with

[2] See Garabedian, Carl A. "Some Simple Logical Notions Encountered in Elementary Mathematics," THE MATHEMATICS TEACHER, October and November 1931.

[3] Young, J. W. "Functions of the Mathematical Association of America," *The American Mathematical Monthly*, January, 1932, pp. 6-15.

pure subject matter or professionalized matter that will not be beyond our secondary teachers of mathematics and let us have such material for publication in THE MATHEMATICS TEACHER.

*Thoroughness and discipline.*—We are behind the leading European countries not only in the extent of our work, and in scholarly methods of approach, but also in thoroughness and in discipline. German teachers who try to teach our boys complain about our lack of discipline and thoroughness. While Germany may have too much discipline we should not be excused for going to the other extreme.

We have continued to excuse ourselves as I have done here by pleading universal education as the cause. We would not need to give this excuse if we made the work of the senior high school elective. We should then be free to follow the lead of our friends across the sea and give our best pupils a course in mathematics that would be stimulating and profitable.

*General mathematics.*—We have tried to remedy some of the defects in our educational scheme by introducing the junior high school course and this has been a noteworthy attempt. However, let us not delude ourselves as to its uniform acceptance. It is received in name only in most schools in this country today. In such schools no genuine reform has taken place.[4] We are today the only important civilized country in the world giving eight years of instruction in arithmetic. This situation should not be tolerated by the mathematics teachers in this country and it would not be tolerated if we were all genuinely interested in reform.

People who are not sympathetic with the general mathematics movement say that the course in mathematics in the junior high school is superficial and this is true. Such an exploratory course must of necessity be superficial because we are not as thorough as the Germans.

In Germany I saw geometry of an informal sort taught in the fourth grade and in the eighth grade of the boy's schools I saw demonstrative geometry taught. The Germans begin the study of geometry early and spread it all the way through the course. The same thing is true of the other branches of mathematics.

The course, in mathematics differs in so far as placement of material goes in the various states of Germany, but one finds usually

[4] Koos, L. V. *The Junior High School,* Ginn and Company, 1927. pp. 168-176 and 244-245.

a more general type of course including algebra, geometry, trigonometry, and so on than he can find here in America.

*Failure to follow our leaders.*—Again, we are slow, not only to take advantage of what is found out by studying the work of other countries, but we do not profit as a group from what the leaders in our own country recommend. Douglass recently said:[5] "It may seem odd that so important a matter as selecting and arranging the materials of instruction should have been attacked with modern research techniques only after such methods had been previously applied to other phases of the scientific study of education. While significant changes may be observed to have taken place in the materials of instruction in secondary schools in the last century, it is not certain that all these changes have made for improvement. It seems quite certain that they have not kept step with changes in the social, economic and scientific developments in American society and with the changes in the character of the school population for whom the curriculum is intended."

To quote Douglass once more: "But another conservative influence is serving as a second and more impregnable line of defense. National uniformity has rendered the American ladder system of education unwieldy and offers passive but solid resistance to change. An excellent example may be cited from the field of mathematics. For twenty years correlated and unified mathematics, including arithmetic, algebra, geometry, and trigonometry, has received approval of the better qualified theorists in mathematical curriculum thinking; yet recent textbooks in junior high school mathematics are little more than two years of arithmetic and one year of algebra. The most recent products of authors who have been regarded as leaders in the movement for unified mathematics have been a keen disappointment to those who had been expecting a series of books patterned after the theories of those authors as expressed in their discussions appearing in THE MATHEMATICS TEACHER and elsewhere.

"Books which involve material departure from the traditional content uniformly obtaining in secondary schools in this country are not good risks for publishers. Publishers are more interested in books that will sell widely. That is natural. Textbooks, the use of which necessitates study and revision of courses on the part of the busy,

[5] Douglass, Harl R. "The Contribution of Research to Secondary School Curriculum Construction." *School and Society*, 32: 411-416.

half-trained teachers, half or more of whom are novices, apprentices in their first five years of experience, do not make strong bids for state adoption and appeal only to a restricted field. Contributing heavily to this inertia is the conservatism of school administrators and of the lay public. Administrators are not uniformly close and careful students of curriculum research and thinking. In large numbers they lack both time and background for judging curriculum suggestions in all fields on the basis of their merits.

"They are loath to abandon beaten paths (on which they, if wrong, are at least not conspicuous) in favor of new theories and practices, until such theories and practices have proved themselves. But there is the rub, the difficulty of proving the ultimate superiority of alleged improved curriculum materials is indeed a baffling problem. The attitude of the public school is much like that of interested alumni towards intercollegiate athletics, interested but improperly prepared to arrive at the judgments which are made so freely and vigorously, the freedom and vigor being in inverse relationship to the orientation and background from which they spring. Innovations, to them, are likely to be thought of as degeneration from the good old standards —the curriculum that 'made' great statesmen, authors, scientists, and college presidents.

"But most unfortunate of all is the lack of a body of scientific curriculum information based upon researches involving reliable and valid data. Proposed changes in the curriculum have largely been subjective and deductive conclusions arrived at by small groups of the 'best minds.' The value of such an approach should not be underestimated. There are many curriculum problems which lie so close to the surface that their solution requires no complicated research procedures, no gathering of bodies of new data and no refined statistical or other intricate methods of reasoning. A clear, keen mind and a rich experience suffice. An example of such problems is the question whether there should be retained in the ninth grade mathematics certain processes in algebra known to be rarely employed and of little or no value in the pursuit of the study of higher mathematics."

### *Depleting, Enriching, and Widening Influences*

*Extramural examinations.*—In addition to the lack of academic background on the part of the mathematics teachers in this country there are other factors that make progress difficult. In the first place,

our extramural examination system tends to subject not only the mediocre teachers, but also some of the better ones to a type of slavery from which there is no escape.[6] The most powerful of these extramural systems are the College Entrance Examination Board and the New York Regents' system. In the case of the latter we have a type of formalism and a relaxation of standards that should not be tolerated. For some in our educational group "justifying relaxed standards has become a virtue." Ninth grade questions of a short time ago constitute the eleventh grade questions of today. It is said that American standards on our tests in arithmetic have to be revised upward for Canadian schools.

President Hutchins of The University of Chicago recently said, "Though schools and colleges should be interdependent, each should be autonomous educationally. College entrance requirements should cease to hamper the forces of experiment and progress latent in the schools. If one thing is clear, it is that the primary purpose of the high school is not to prepare students for the colleges and universities. By behaving as though it were, the college and universities repress the high schools, and to that extent weaken themselves by weakening the educational system."

In speaking with one of the prominent school superintendents in this country about progress in mathematics he said, "I believe in the more progressive type of mathematics course as much as you do, but so long as the pupils under my jurisdiction have to pay allegiance to the College Entrance Examination Board and the New York Regents' System I do not expect to do anything about establishing a more forward-looking course."

Moreover a prominent principal of one of the New York schools told me recently that her teachers of mathematics were slaves to the New York Regents' system and that there was little hope of getting such teachers interested in any kind of experimentation as long as their pupils had to pass the Regents' examinations.

*School administrators.*—In the second place some of our school administrators in this country have too much power or at least often abuse their power in the selection of teachers, the building of courses of study, the choice of textbooks, and the like. In spite of the large number of children going to high school today the need for teachers

[6] Reeve, William David. *Fourth Yearbook of the National Council of Teachers of Mathematics,* pp. 131-142.

of mathematics is not so urgent nor teachers so scarce that a school superintendent or high school principal is justified in giving a class in algebra to a football coach whose training to teach mathematics is almost nil, or to a second rate teacher who may be good-looking. Moreover, he has no right to force a certain textbook on a teacher or on a group of teachers against their wishes. Such a situation as I have described above would not exist in Germany where teachers of mathematics are not only free to make their own courses of study and choose their own textbooks, but also have a large hand in setting what we call the final examination. In fact, the study of mathematics in Germany is looked upon by almost everybody as second in importance only to the mother tongue and school administrators have no such power as they have in this country.

*Influence of the general educators.*—A third factor retarding progress is the attitude of certain professors of education in our colleges and universities who for one reason or another do not back us up in our attempt to lift the academic requirements of those who intend to teach mathematics. They tell our student teachers that if one is to teach mathematics in the high school, all he needs to know is high school algebra and high school geometry. Again, one finds better cooperation in the German schools. Some of our general educators like Professors Judd and Bagley[7] are notable exceptions. Theirs is a fundamental concern for American education. My quarrel is with the obstructionists who at a time when cooperation is needed between the educational generalists and the subject matter specialists persist in rocking the boat.

The "subject matter scholar," as Professor Bagley calls him, has been mostly absent from our councils on curriculum construction for the last generation. I believe with many others that it is a bad thing for the educational generalists to be in absolute charge of curriculum construction. In some cases they are not only building the curriculum, and writing the textbooks in mathematics as well, but in a few cases are beginning to preside at the sectional groups of teachers of mathematics in State Association meetings.

*Further conditions in Germany.*—In order to give you some idea of what I saw in Germany let me begin by telling you what I saw in Göttingen, Hannover, Hamburg, Lübeck, Magdeburg, Halle, Kas-

[7] Bagley, William C. "Professors of Education and Their Academic Colleagues," THE MATHEMATICS TEACHER, May, 1930, pp. 277-288.

sel, Frankfurt, Stuttgart, Düsseldorf, and Essen. I visited both public and private schools, schools in the city and schools in the country. I saw good teaching and I saw bad teaching, and I may say here and now that I think the Germans are no better teachers than we are. That is, they have no better knowledge of children, have no finer classroom spirit, and try no harder to give the best that they know to the situation.

Where a German teacher of mathematics was inefficient I found the pupils as ready to take advantage of the situation as our American children, but I saw very little "horse-play" anywhere largely because teachers usually had something ready to do and proceeded to do it as soon as the recitation began.

The favorite type of procedure as I saw it was the plan of sending a pupil to the board to do a piece of work while the discussion between the teacher and the class proceeded. Rarely was a boy who was not prepared permitted to remain long at the board.

In lettering geometric figures the Germans generally used Greek letters, a plan we might well adopt in this country. I can see two arguments for the method to one against it.

One reason why the German teachers of mathematics cover so much more ground than we can is due to the fact that about eight per cent of their youth are in the secondary schools while we find about fifty-eight per cent in similar schools here. However, I feel confident that the most gifted children in our secondary schools are equal in native ability to the students of the same age in Germany.

It was interesting to note that there was scarcely a mathematics classroom in Germany that did not have access to squared board, colored chalk, mathematical instruments of various kinds, and all sorts of models to use in illustrating the work. Each department of mathematics usually has a workshop where the pupils make models or other useful material. Such material is unknown in many of our American classrooms.

The ventilation in most of the classrooms I visited was terrible. It may be as bad over here, but I doubt it. Outside of the purely mathematical aspects the general comfort of the rooms was not up to our American standards. The seats were generally old fashioned and not adapted to the age and size of the pupils. In some respects one can see still the influence of the former military régime. For example, every time I entered a classroom in Germany all of the pupils stood at at-

tention immediately. In most cases they repeated this performance when I left. In general, however, one can see a great deal of freedom and the give and take between teachers and pupils is far greater than one would have seen before the war.

The German teachers do not make use of modern testing devices as we do in this country. One sees little effect of the recent testing movement on the classroom work in Germany. This does not mean that searching tests are not given at certain intervals, but it does mean that they do not carry on such detailed diagnostic testing as is done here. Perhaps it is not so necessary with the more gifted type of pupil one finds in the German secondary school. The examinations that I saw which were given to boys at the end of their course in the *Oberrealschule* were very comprehensive and difficult. Most of the results that I saw would do credit to an American student in the last year of college work.

*Recommendations.*—If we are going to improve the mathematical situation in this country, the following suggestions may be helpful:

1. Two types of scholarly men and women should be developed. First those whom Professor Bagley calls *research scholars* whose major interest is to expand mathematics vertically. Second, those whom he terms *teacher scholars* and who are primarily interested in the horizontal expansion of the subject.

2. We need courses in methods and in professionalized subject matter for students who intend to be college teachers of mathematics. The teaching of college mathematics in many places is far below the standard.[8]

3. There is no question that salaries of teachers in this country should be raised so as to attract capable men and women who will remain in the teaching profession, but when that is done, we shall have to raise our academic requirements as well if we are to bring about genuine reform. Already the increasing number of college graduates with good academic background in mathematics who are enrolling in our teacher training institutions in America augurs well for the future. With an oversupply of candidates for teaching positions we ought to be able to raise our standards and employ only the best of all those who are available.

[8] Seidlin, Joseph. "A Critical Study of the Teaching of Elementary College Mathematics." Bureau of Publications, Teachers College, Columbia University. Contributions to Education, No. 482.

4. The work in mathematics and physics in Germany is better correlated than in America because of the fact that the mathematics teachers in Germany can also teach physics.[9]

It seems to me that we might require some training in science, at least in physics, of all prospective teachers of mathematics. In some schools in America the work in mathematics and physics is already being correlated.

5. We must co-operate with subject matter specialists in other fields in demanding more recognition on the programs of general educational meetings like state associations, sectional meetings, teachers institutes, and the like. Some state associations are so completely under the control of secretaries (often men of meager academic training) that it is almost impossible for a subject matter group to secure any outside talent without digging down into their own pockets to secure the necessary funds. These same subject matter groups contribute regularly to the support of these state associations and are often compelled to sit and listen to some "spellbinder" speaking on a topic that is foreign to every genuine interest of the subject matter specialist. There is no reason why some topic in one subject should not be ably discussed each year before the general group at every state meeting in the land. We will have better results in American education when various groups concerned understand a little better and have a little more sympathy with the other person's point of view.

6. For the benefit of my own group, the National Council of Teachers of Mathematics, may I suggest that we can get nowhere by passing resolutions on what we do or do not believe. We need action. Certain unsatisfactory conditions that exist at present such as the static condition of certain subjects like geometry, the question as to whether we should or should not have a one year course in plane and solid geometry cannot be settled by wishing if they can ever be settled at all.

7. We need now as never before to get all teachers of mathematics from the elementary school through the university to co-operate in solving our common problems in order that there may be no waste effort and in order that we may serve the best interests of the pupils whom mathematics is to serve to the end that we may offset the re-

[9] Congdon, Allan R. "Training in High School Mathematics Essential for Success in Certain College Subjects." Bureau of Education, Teachers College, Columbia University. Contributions to Education, No. 403.

actionary influence due to the educational generalist referred to above.

John Dewey recently said, "Teachers have been too passive, too submissive, to the dictation of outside powers. It is our part to maintain the intellectual independence of the educative process and to strive for the right to present the defects as well as the excellencies of the existing economic order even if by so doing some interests are offended."

8. We shall doubtless do considerably better work in mathematics when we resort to homogeneous classification of pupils provided we subsequently teach as though the classification has been previously made. However, we shall gain nothing by classifying pupils into homogeneous groups and then going on teaching them as though there were no individual differences in ability. Teachers and administrators alike must become better informed as to the best practice in homogeneous-ability grouping.[10]

9. As fast as we improve the teachers of mathematics we can lift the level of the articles in THE MATHEMATICS TEACHER and in the National Council Yearbooks. However, we shall still face the problem of getting these publications in the hands of teachers where they will be read. While it may be a surprise to some of you, there are plenty of mathematics teachers who do not know that there is such an organization as the National Council of Teachers of Mathematics.

We ought to have a membership of 10,000 in the National Council of Teachers of Mathematics. We have doubled our membership in the last five years. If we double it again in the next five years our goal of 10,000 will be more than reached. I hereby pledge with you my interest, my strength, and the fullest co-operation to the accomplishment of that goal.

[10] Coxe, Warren W. "Our Homogeneous-Ability Grouping Confusion." *Journal of Educational Research*, January, 1932, pp. 1-5.

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## National Council Yearbooks

BE SURE that you get your copies of the various National Council Yearbooks before it is too late. The first is already out of print. See page 245 for titles and prices of the others.

# Practical Application of Mathematics to Engineering Problems*

*By* HARRY COLE, *Structural Engineer, Architects' Department Board of Education, Chicago, Illinois*

AS WE LOOK BACK in ancient times, we sometimes wonder how it was possible for the Romans to build that great aqueduct of Rome, their many beautiful and massive edifices, and the bridges in Italy and Gaul without the aid of mathematical analysis or formulas! The same, being true with all ancient culture, the Arabs, the Greeks, the Hindus, and the Chinese, with all their beautiful and massive structures, some of which are standing today, give us the feeling that their architects and engineers had the uncanny feeling of mathematical insight. Through the middle ages many buildings and engineering structures were erected partly with the aid of mathematics and partly with the knowledge and experience of great architects and builders, who certainly had a *wonderful* sense of proportion *and* a definite insight of the *forces* of nature without the reasons therefore. And yet the buildings and structures stand today as mute testimony of the wonderful capacity and ability of the engineers and builders of past ages.

It is true that the principles of algebra, geometry, and trigonometry were well known to the ancients and to mathematicians of the middle ages, but it was not until the last century that the scientists were able to apply these well known principles to engineering *structures!* Not until this present era of industrial development, namely, within the last hundred years, was it possible to calculate by means of mathematical equations, expressed in terms of physical laws, the internal stresses and strains of the various building materials, thus enlarging upon the field of industrial development and construction a thousand fold.

It might be truly said that the entire phase of modern progress is largely due to the contributions of the mathematicians of former ages in the fields of pure mathematics and its later adaptation by the scientists of the present age to the physical laws of nature, thereby

* Read before the Chicago Men's Mathematical Club.

combining the necessary factors handed down to the present day engineer, in the solution of engineering problems.

The problems of the engineer are many, intricate in all its phases but definite in scope for his investigations. The engineer employs from the very beginning, mathematical analysis of his problem. Certain theoretical assumptions are made, based on fact and experiment and a layout is made of the particular problem he has in mind. This problem is then carefully studied in detailed form and calculations or computations made for every feature of the structural design, in the determination of the proper details of the required structure. With this information, drawings of the engineering structural design are executed and drawn, specifications written and blue prints made for the construction program. This may seem like routine work to the average person, but to the engineering designer no work is more fascinating than to see the realization in actual form of the building or structure of his creation and design.

In view of the fact that modern practice demands a high degree of efficiency, speed and accuracy in the execution of engineering plans and specifications, many new methods have been developed in recent years to aid the engineer in his calculations and analysis of his particular problem. The slide rule is now the inseparable companion of the designing structural engineer. Multiplication, division, squares, cubes, and roots are easily obtained by a slight manipulation of the slide rule and a quick result obtained with very little effort, accurate to three significant figures, which is as close as may be required in most engineering calculations. The 20 inch slide rule and the cylindrical Thatcher slide rule will give results to 4 or 5 significant figures.

In many problems more precise results are required and in that case the Logarithm tables are employed and better still the calculating machine, many types of which are on the market today. It is no trick at all to multiply and divide large figures with results to seven or eight places by means of the electric automatic computing machines, now being used in many offices.

### 1. *Elimination of Errors*

To students, scholars, and teachers of the many branches of mathematics now being employed in the solution of engineering problems, it may be interesting to note the facility and ease with which such problems are taken in hand by an engineering organization and

brought to a successful solution. It must be appreciated by you as scholars of mathematics, that organization of talent and co-ordination of effort must be secured in every engineering office in order to get the required result in the limited time allotted for a complete engineering design. Sometimes a complete structural design of a forty-two story building is required within a month's labor. The plans must be completed within this time limit, specifications written, all finished and ready for construction. Contracts are let and building erected within one year of the beginning of construction as you have noticed everywhere in the large cities of this country, during the last decade particularly. You will rightfully ask and ponder: "How is this possible! In such an absurd short time limit! Also why was it not possible to accomplish this very intensified building program in former years?" My answer is—Mathematics! Yes! Mathematics!—but applied mathematics with the highest degree of organized and systemized effort and ingenuity. Unfortunately, many of the members of the engineering profession know very little about higher mathematics, much less in its application for systematic and quick solution of their particular problem in hand. For that very reason, one of the greatest worries and unfortunate grievances in every office is the occurrence of numerous errors in designing work: (1) Unavoidable errors due to poor judgment, (2) Trivial errors, (3) Serious errors. These errors, unlike the errors in an examination at school, are at times very dangerous and may cause a failure in the structure with loss of life as in the case of the Quebec Bridge. This bridge as you know, failed during construction, largely due to the lack of proper engineering design and poor mathematical analysis. It is not the intention of this paper to go into the many failures in this country due to poor mathematical analysis, but merely to present a method of minimizing the typical errors in an office or perhaps eliminating them entirely. Just as in the business world, when a corporation is formed and stocks issued, the brokers and bankers will apply the acid test, so must we employ such methods in every organization of engineers during the course of an engineering design. An engineer, who is very well grounded in mathematical principles will always check his own work for errors, by employing a method of analysis other than the one he first employed and see if he gets the same result. This will assure him of the correct answer providing his assumptions were correct in the first place.

Let me cite a concrete example so that you can understand what I am driving at. In the design of a viaduct, such as is being built over the Illinois Central Tracks at Randolph Street in this city, at the present time or a portal to a bridge or some similar structure, there are at least three different known methods of mathematical analysis. One of these methods is the slope deflection method, the other is the method of Least Squares and the third is the approximate method derived by some German engineers. All of these methods will give practically the same required result with sufficient accuracy. Therefore there is no excuse for any engineer to derive at a wrong result in design. Unfortunately, as I stated heretofore, too few engineers avail themselves of the many known ways in solving a problem. For their benefit every office has a checker, whose duty is to check the plans independently, also the design submitted by the original designer. Mistakes are finally combed out in this fashion. In that way the acid test is applied to every detail of structural design so that no mistakes will occur on the plans, which may surely cause a lot of trouble on the job if allowed to pass. Of course some mistakes are more serious than others, but to the good designer all mistakes are serious and therefore must be eliminated. (The trivial errors may be combed out in the final analysis, but the serious errors are discovered only after a failure of the structure.)

### 2. *Short Cuts in Calculations*

Our next subject of concern is to provide means of attaining speed on the great amount of routine work which is prevalent in all engineering offices on major engineering structures. Of course, we are not going to sacrifice *accuracy* in order to get the work out in time. Much can be said on this phase of the subject, but it will suffice to give a few brief examples in order to bring out my thought on the subject. In the point of view of the engineer in charge of the job, he can secure varying degrees of speed according to his own experience in the handling of the men *under* him during the course of preparing the plans.

Let us assume that a foundation must be designed, such as was built for the new Lane Technical High School in the northwest section of the city of Chicago. The site there required a wood pile foundation, due to the fact that the material at the site was dumped refuse, and entirely unsuitable for bearing a building of such magnitude as the

Lane Tech. There were over 5,000 piles driven down to hard bearing in clusters of 4, 5, 6, 7, and 8 piles each, supporting the columns of the building frame, upon which the architectural finish is supported. In the office, the loads superimposed on each and every one of the 500 or more columns are figured out carefully, checked, and the foundation designed. Standard forms were made out for the various loads and the proper figures inserted over and over again in the proper space provided on sheets of paper so that the designer with the least amount of effort or thought mechanically sets down the right figures, compiled in proper form and each cluster of piles and the concrete mats and piers designed by means of tables previously computed, so that simultaneous action can be secured in design in the least possible time. Where there are two or three men working on the same problem, coordination of effort is most essential and necessary for a successful design. Most of this work as you see, is merely routine and mechanical after the system is once worked out. The slide rule is entirely used in all the calculations for this work, figures set down in tables, the designs made and repeated until the whole work is then completed. The checking or acid test comes later. The essential part of this particular phase of the subject, is the securing of speed in design with the least amount of effort. I may add that engineering tables, standards, mathematical graphs based on formulas and special slide rules may be employed to facilitate the work. It must be definitely kept in mind that all these forms or tables must be absolutely correct and familiar to the designer. Even with all these precautions, mistakes may occur but with clear and conscientious work the errors are brought to a minimum. Also, these various forms as described are based upon mathematical formulas and equations, which have been worked out for us by the knowledge and experience of the scientist—mathematician and engineer who had come before us and contributed this knowledge to us.

The schools and colleges have grounded the theoretical side of the engineer, and in practice he slowly learns to apply his knowledge of mathematics to his individual problem as his experience broadens. A thorough knowledge of mathematics trains the mind to think logically and clearly so necessary to all engineers, just as philosophy does to the philosopher or student of culture. Mathematics is comparable to the study of philosophy in that it employs reasoning capacity for the conclusion of an analogy or hypothesis.

## Formulas

I would like to cite some very interesting engineering formulas that are sometimes employed in the solution of engineering problems with a brief description of each formula and the routine of handling them in actual practice, in order to convey to you the mathematical significance of each operation.

The formula $v = C\sqrt{rs}$ is used in determining the flow of water in open channels. This formula is known as the Chezy's formula and contains a number of variables. The letter $(v)$ in this expression is the velocity of the water flowing downstream expressed in feet per minute. $C$ is the coefficient based on the type of channel considered and kind of underlying material. To solve for this value Kutter gives us the expression $C = \dfrac{41.65 + \dfrac{0.00281}{s} + \dfrac{1.811}{n}}{1 + (41.65 \dfrac{0.00281}{s}) \dfrac{n}{\sqrt{r}}}$ based on experimental data, being the coefficient of roughness for the underlying material such as timber, gravel, sand, clay, or stone, varying from $n = 0.009$ to $n = 0.035$, and $r$ is the hydraulic radius which is a measure of the area of the channel divided by the wetted perimeter and $s$ in the original formula is the slope of the channel expressed in inches per foot. After determining the various quantities in the formula, the values are substituted and the velocity obtained. It would indeed be a lengthy operation if we had to figure all these values out long hand, but the slide rule comes to our rescue and it is very simple and easy. In actual practice a form or table is made for the different values and the designer needs only to substitute the proper figures for his particular condition. In that way office routine is cut down to a minimum. The question will arise, whether these formulas are just empirical expressions or based on theory. The answer to that question is that no reliable engineer will accept a 100 per cent empirical formula determined by others for his design, and will demand a formula based on theory and experiment, largely theory. It is the duty of the engineer to seek the truthful expression for his most responsible work because so much depends upon his final design. The Chezy formula was derived by means of mathematical expressions, representing the frictional resistance per unit area of rubbing surface finally expressed

in terms of fluid. In determining the flow of water, we have other means to check our work, such as the measuring of the flow over a weir or the use of a velocity gage. The results should check within 5 per cent and our work is considered correct. The engineer today no longer uses the so-called rule of thumb method, so prevalent fifty years ago, but he has at his disposal numerous theoretical expressions which are absolutely accurate and will stand all tests of nature.

Let us now turn to the determination of an arch! There are several methods in tackling that problem. One of the most common is the graphical method of drawing the medial line of resistance of the arch ring and fitting the intradas and extradas arcs and building up an arch depending upon the material. There are many notable examples of concrete bridges with spans from 100 feet to 200 feet both in this country and in Europe, built within the last hundred years. There is also a theoretical method of determining the arch ring, known as the elastic theory, which is very complicated in its mathematical form. The expressions for the unknowns are derived through methods of calculus making use of the theorem of least work, stated as follows: "The internal work done in any stationary structure by the application of outer forces will be the least possible, consistent with equilibrium," and expressed as

$$W = \int \frac{M^2}{2EI}\,ds + \int \frac{T^2}{2AE}\,ds$$

Solving and applying this equation for the arch ring, the three equations of equilibrium are derived and in that way the arch is solved. By this method several weeks may be required to solve a single arch, the work is very tedious and difficult due to the cumbersome and unwieldy form of the expressions in order to use the exact method. In actual practice some approximations are made thus cutting down the work, and yet keeping the results within reasonably accurate values. During the ancient times the Romans understood the practical application of the forces on the arch, but they had no way of determining the true arch as we know it today. But we are still very far from knowing all the action of loads and forces on the arch and much study and investigation must be made to derive better results and more accurate analysis under all conditions for future designs. In other words, we are still studying the arch analysis and must turn to you again as mathematicians for mathematical expression.

There is one subject that has been very interesting to the engineer

in recent years, and that is wind investigations and stresses in tall frames. Much has been written on this subject, but we are far from the ultimate solution of this problem. You have probably heard that some of the tall buildings in New York sway in the wind so greatly that one gets seasick on the top floors. That is due to the lack of wind bracing and general stiffeners of the entire frame. The Woolworth Tower has been observed to move 18 inches each side of the vertical line during a gale of sixty miles per hour, making a complete cycle in a minute and a half. The tall building towers in Chicago generally have more and heavier wind-bracing members than those in New York, with the result that the movement is much less. We have at our disposal a practical method of analysing wind stresses in tall building frames, but are still seeking far more accurate and more reliable mathematical expressions to solve the problems of our ever growing skyscrapers of this country. As before, I am sure mathematics will again be our salvation and soon we will be able to solve this very problem, based purely on theoretical considerations.

I mention these facts in order to bring out how closely allied the engineer, who designs structures, is to the mathematician, who teaches and develops mathematical expressions on theoretical bases.

### *Personal Equation*

One of the most useful subjects of mathematics as applied to engineering problems in the office is the precision of measurement, with a good knowledge of the Method of Least Squares, the Theory of Probability and expression of the Personal Equation of the designers working on engineering plans. Of course this phase of the subject concerns the engineer in charge of the job directing a number of men under his supervision. In applying the final check or acid test, he must use his judgment regarding the accuracy of the results by the methods used in design, and weigh the probability of an error due to lack of repetition of application, and also the probability of an error due to the personal characteristics of the men under him. All this he will weigh with methods based on theory as well as judgment. The engineer in charge is the final judge. It is interesting to note how this practice differs in the various offices, but this being a human problem of vital interest, it is the most essential factor in the solution of this particular problem. I mention this fact only in passing with the hope that more data on this subject will be studied for the benefit of the engineering profession as well as the mathematician.

# The New President of the National Council of Teachers of Mathematics

MR. WILLIAM BETZ, the new President of the National Council, comes from a family in which teaching has been the principal occupation for four generations. True to this family tradition, he began to teach while still in his teens. Having majored in the classics, it was his original intention to devote his life to the teaching of Latin and Greek. The fates led him along a different path. The first position which was offered to him was that of a "supply" teacher in mathematics in the Rochester High School from which he was graduated. Somehow, he never got away from mathematics after that. An apprenticeship service of a comparatively short period seemed sufficient to convince Principal A. H. Wilcox that it was safe to appoint this young man as head of the department of mathematics in the East High School. Years of strenuous work became necessary. Three European trips gave to the young teacher a first-hand acquaintance with schools abroad and enabled him to get a personal glimpse of the influence of such leaders as Prof. Felix Klein at the University of Göttingen. Several years of postgraduate study were devoted primarily to the foundations of geometry and to applied work in physics.

When Rochester began to develop its system of junior high schools, Mr. Betz was asked to undertake the experimental determination of the new courses in mathematics, with the able assistance of a group of teachers who had been specially selected and trained for this task. The work which was begun in Rochester under his direction has since become generally known through his lectures, his summer courses, and his various publications.

A few years ago Mr. Betz was chosen to guide the curriculum revision work in the senior high schools of Rochester. He was also appointed specialist in mathematics for all the public schools of Rochester. In addition Mr. Betz has been a lecturer in Education at the University of Rochester. While he has been honored by being made vice-principal of East High School, he has regularly devoted much of his time to the training of teachers in service, and to experimental or demonstration teaching in connection with vexing questions of the curriculum.

Mr. Betz has served as a member of important mathematical com-

mittees. He has been identified for years with the progressive movement in the teaching of secondary mathematics. He has had a large share in the preparation of the new mathematical syllabuses of New York State. He has lectured before many audiences of teachers in the East and the Middle West, both on mathematical topics and on general educational problems. He has given summer courses in such institutions as Cornell University, the Cleveland School of Education, and Teachers College, Columbia University.

Having been one of the charter members of the National Council, Mr. Betz has been one of its most loyal supporters. His contributions have appeared in the columns of THE MATHEMATICS TEACHER and in three of the Yearbooks. On another page of this issue will be found a personal message from the new president, to which we invite the attention of our readers.

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THE MATHEMATICS TEACHER
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# A Message from the New President of the National Council

*By* William Betz

Just two years ago my worthy predecessor, Dr. John P. Everett, in looking back upon the development of the National Council during the past ten years of its existence, wrote these significant lines:

*The National Council exists not to perpetuate a régime, but to afford a medium for the expression and exercise of the highest type of co-operative endeavor in an attempt to discover what the best interests of youth and adulthood demand and what are the best methods of serving those interests.*

This ideal attitude, I am sure, has guided the official life of the Council ever since its modest beginning at the Cleveland meeting in 1920. I regard it as a high honor and a great privilege to have been chosen by my colleagues to serve so noble a cause and to "carry on" in the spirit of such a splendid tradition. It will be my constant endeavor to maintain the standard of excellence which has so uniformly characterized the work done under the auspices of the Council.

And now, let me take this earliest opportunity of outlining some of the tasks which appear to lie ahead of us in the near future. It is my earnest desire, because of the magnitude of these tasks, to enlist on their behalf the interest and support of all our members.

There is a distinct feeling in our ranks that the Council is now prepared, owing to the able leadership it has enjoyed, and because of its growing membership, to attack more directly certain fundamental issues that affect the very existence of secondary mathematics. The Detroit meeting of last year, and still more the recent meeting at Washington, seemed to give to the Board of Directors a definite mandate for a policy of aggressive action with reference to the baffling problems that have been thrust upon the school in recent years.

It is hardly necessary to rehearse these transforming influences in detail. There is a general feeling that we are at the end of an era and that our schools are groping more or less blindly toward new horizons. Old landmarks are disappearing and uncharted territories are being opened on all sides. Everywhere the slogan is "Change!"

Mass education seems to be here to stay. As never before, we are wrestling with the all-inclusive problem of formulating for our Ameri-

can schools a philosophy of education that is not based on the popular slogans of the hour, but on the bedrock of "first principles." Perhaps the most crucial question which we are facing as teachers of mathematics is this: *Precisely what is the real or potential contribution of mathematics in the lives of our young people, of whom a large number may never have occasion to use secondary mathematics for vocational purposes?* On our answer to that question will depend, in a large measure, the future of secondary mathematics in our schools.

We shall not be permitted to evade this issue. Many factors are combining to force a solution upon us. Abraham Flexner reminded us years ago that nothing should remain in the curriculum of a modern school for which a "constructive case" cannot be made out. Social science, natural science, the fine or applied arts, and a multitude of "activities" are all struggling for a larger share of the pupil's time. It has become one of the major sports of the psychologist and the general educator to dispute, not always in polite form, the time-honored cultural claims of mathematics, in spite of weightly scientific evidence in favor of "transfer." If to this be added the well-known fact that mathematics is regarded as a high-mortality subject, the story is complete. And so, the "entrenched" position of mathematics is now being attacked on all sides.

What shall we do about it? A passive attitude will only aid the negative propaganda emanating from the ranks of educational iconoclasts and self-styled apostles of the "new education." The Board of Directors of the National Council thoroughly believe that mathematics can and should face any open and fair investigation of its merits. Its age-old record of service, both practical and cultural, cannot lightly be ignored or set aside. The universe in which we live is incurably mathematical. Our modern civilization is becoming more mathematical from day to day. To rob our adolescent youth of an acquaintance with the very web of this industrial and scientific age, is to send them out hopelessly handicapped in their endeavor to understand the world in which they move and have their being.

Now, to cope with the vexing situation suggested above, the Board of Directors has authorized the organization of three new standing committees. *First,* there is to be a Committee on Individual Differences, which is to investigate the merits of ability grouping, of differentiated curricula, and the like. *Second,* a Committee on Geometry is to extend and make generally available the practical studies which have been going on in this difficult field for some time. *Third,* there

is to be a Committee on "Policy," which is to attempt a new synthesis of the most authoritative views concerning the educational significance of mathematics and of the guiding principles that should govern the work of the mathematical classroom.

As soon as these committees have been organized, the chairman of each group will present in the columns of THE MATHEMATICS TEACHER a preliminary statement of the program to be followed in each case.

A circular letter, outlining in provisional form the problems to be investigated by each committee, will be sent to each of our affiliated organizations. The officers of the Council are very anxious to obtain from these organizations, and from each member of the Council who may be personally interested in any phase of the investigation, helpful suggestions with reference to the membership of each committee as well as the most promising modes of procedure.

Allow me to close this brief message by expressing the hope and the conviction that this new co-operative service which the Council is ready to sponsor will have the same united support that has been so generously given by its members throughout its entire career.

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## A Tribute to the Unknown Teacher

And what of teaching? Ah, there you have the worst paid, and the best rewarded, of all the vocations. Dare not to enter it unless you love it. For the vast majority of men and women it has no promise of wealth or fame, but they, to whom it is dear for its own sake, are among the nobility of mankind.

I sing the praise of the unknown teacher. Great generals win campaigns, but it is the unknown soldier who wins the war.

Famous educators plan new systems of pedagogy, but it is the unknown teacher who delivers and guides the young. He lives in obscurity and contends with hardship. For him no trumpets blare, no chariots wait, no golden decorations are decreed. He keeps the watch along the borders of darkness and makes the attack on the trenches of ignorance and folly. Patient in his daily duty, he strives to conquer the evil powers which are the enemies of youth. He awakens sleeping spirits. He quickens the indolent, encourages the eager, and steadies the unstable. He communicates his own joy in learning and shares with boys and girls the best treasures of his mind. He lights many candles which, in later years, will shine back to cheer him. This is his reward.

Knowledge may be gained from books; but the love of knowledge is transmitted only by personal contact. No one has deserved better of the republic than the unknown teacher. No one is more worthy to be enrolled in a democratic aristocracy, "king of himself and servant of mankind."

—HENRY VAN DYKE

# Trigonometry in the High School[1]

*By* H. W. Bailey, *University of Illinois*

*Introduction.*—The University of Illinois has had for many years a two hour course in trigonometry, known to several thousand present and former students as Mathematics 4; the prerequisites are one unit of algebra and one unit of plane geometry. In the fall of 1924 there was introduced a two hour course in advanced trigonometry, called Mathematics 5. The prerequisites for this course are one and one-half units of algebra, one unit of plane geometry, one-half unit of solid geometry, and one-half unit of trigonometry or Mathematics 4. The experience of the department with this latter course forms the basis of my remarks.

I wish to make it perfectly clear that I am looking at the high school work in trigonometry from the standpoint of one concerned with college mathematics who has to deal with the high school product. I am very desirous that none of my remarks be interpreted as an endeavor on my part to say what the high school should or should not do in regard to its mathematics curriculum as a whole. I do not consider myself sufficiently well acquainted with the peculiar problems of the high school to criticize them. I hope that this reservation will be kept in mind in connection with everything that I have to say.

*Mathematics 5.*[2]—Mathematics 5 has been offered in each fall semester starting with the first semester of 1924. For the first four years, 1924 to 1927 inclusive, no provision was made to care for a poorly prepared student. In effect the department went on the assumption that if a student had had a course in trigonometry in high school he was competent to carry this course in advanced trigonometry. Four years was ample time to demonstrate that this was a hypothesis contrary to fact. During this period nearly every section had so many poorly prepared students that scarcely as much as twenty per cent of the time was spent in advance of the work covered in Mathematics 4.

In the fall of 1928 a change was made in the administration of

[1] Read before the Mathematics Section of the High School Conference of the University of Illinois at Urbana in November, 1931.

[2] This course was organized under the direction of Dr. C. F. Green who was in charge of freshman mathematics courses until November, 1929.

the course. This year registration in each section was allowed to go to approximately forty. Each instructor was given an outline of material to be covered each day for the first five recitations. At the sixth recitation an hour examination over the material covered in these first five recitations was given to all students. The papers were graded in committee and on the basis of this work the poorest students in each section were shifted back to a section of Mathematics 4 to repeat the course in plane trigonometry. The number shifted in each section depended on the enrollment in the section and the number of places vacant at that hour in sections of Mathematics 4. Most of the remaining students completed the work in Mathematics 5.

Essentially the same plan was carried out in 1929. The examination was changed somewhat but it remained of about the same degree of difficulty and length.

In these two years all sections covered complex numbers and most of the work in spherical trigonometry in addition to a review of plane trigonometry. The average number of failures was also cut by twenty-two per cent despite the increased amount of material covered.

In 1930 for various reasons there was a decided drop in the number of registrations in Mathematics 5. It did not seem possible to use the plan of the previous two years because of the size of the sections in Mathematics 5 and the large size of the sections in Mathematics 4. Consequently students who asked to be transferred back to Mathematics 4 because of faulty preparation were so transferred, but the remainder were left in Mathematics 5. Again in 1931 the registration did not come up to expectations in this course and the plan is not in use this year, though all preparations had been made to use it. By cutting down on the number of sections in Mathematics 5 it is the expectation that the plan can again be used next fall.

*Examination and Results.*—The hour examination given at the sixth recitation was mimeographed on legal cap with room for the working out of the problems and ample margins to be used for scratch work. It was made fairly long purposely in order to get a wide spread of grades.

The seven questions dealt with the following topics in 1928 (as noted above the 1929 examination was much like this):

1. Construction of an angle from the value of one of the functions and the determination of the values of the remaining functions.
2. Numerical values of functions of special angles.
3. Reduction formulas.

4. Expression of all the trigonometric functions in terms of one of them.

5. Line values in the first quadrant.

6. Identities.

7. Derivation of the formula for the cosine of the sum of two angles.

It may be remarked that while the examination appears a little too long the author recently worked it out completely in less than eight minutes. This leaves a little leeway in a fifty minute period using a student-instructor factor of six; by "student-instructor factor" I mean the ratio of the expected time for the average student to do the work to the time required by the instructor.

The grade distribution for each of the two years 1928 and 1929 and the composite is shown in Table I. It should be remarked that each question was graded on the basis of 10 and the grades made up simply by adding, so that 70 is the highest mark possible.

TABLE I

| *Range* | *1928* | *1929* | *Composite* |
|---|---|---|---|
| 0-10 | 28 | 22 | 50 |
| 11-20 | 28 | 28 | 56 |
| 21-30 | 35 | 38 | 73 |
| 31-30 | 47 | 47 | 94 |
| 41-50 | 30 | 40 | 70 |
| 51-60 | 20 | 24 | 44 |
| 61-70 | 14 | 19 | 33 |
| Total | 202 | 218 | 420 |

If one graphs the distribution for each year and the composite, it is evident that the curve for 1928 is decidedly skewed to the left, that for 1929 slightly so, and the composite measurably so.

It is of interest to examine the records of each section separately. Table II gives this material for 1928, Table III for 1929.

TABLE II

| *Section* | *Enrollment* | *No. transferred* | *Ave. grade transfers* | *Highest grade transfer* | *Section average* |
|---|---|---|---|---|---|
| 1 | 38 | 9 | 7.8 | 14 | 27.3 |
| 2 | 41 | 9 | 14.2 | 23 | 34.7 |
| 3 | 35 | 9 | 5.2 | 11 | 27.7 |
| 4 | 27 | 3 | 2.3 | 6 | 41.8 |
| 5 | 42 | 16 | 12.3 | 21 | 27.9 |
| 6 | 42 | 8 | 17.7 | 25 | 35.9 |
| Total | 225 | 54 | 11.1 | 25 | 32.2 |

TABLE III

| *Section* | *Enrollment* | *No. transferred* | *Ave. grade transfers* | *Highest grade transfer* | *Section average* |
|---|---|---|---|---|---|
| 1 | 37 | 11 | 17.3 | 24 | 37.3 |
| 2 | 36 | 8 | 11.6 | 20 | 34.9 |
| 3 | 43 | 15 | 5.1 | 14 | 23.8 |
| 4 | 35 | 9 | 12.1 | 20 | 35.9 |
| 5 | 34 | 8 | 22.0 | 29 | 40.4 |
| 6 | 34 | 4 | 17.8 | 25 | 39.2 |
| Total | 219 | 55 | 12.9 | 29 | 34.8 |

The discrepancy between the total number of students in Table II (or III) and in Table I is accounted for when withdrawals from the course during the first three weeks and absentees from the examination are added in.

*Interpretation of results.*—What shall we say of the high school teaching of trigonometry after looking at these results? One should keep in mind that the author has in addition to them the benefit of five years of additional experience with this course on the part of the department, has taught the course himself at three different times (1924, 1927, 1928), and has made it a special point to talk to the instructors involved the past three years.

Let us review the facts. Each student taking this course had credit for one half unit of trigonometry from high school or Mathematics 4, the number of the latter being comparatively small. He had in addition credit for three units more of high school mathematics. He was given a review over certain special topics, most of which at least should be covered thoroughly in any beginning course in trigonometry. He was then given an examination over these topics and these alone. It was found desirable and possible to transfer approximately one-fourth of all students in this course back to Mathematics 4 which presupposes no previous knowledge of trigonometry; the figure was twenty-four per cent in 1928 and twenty-five per cent in 1929. In 1928 the highest grade made by any transfer was 25 on a basis of 70, in 1929 it was 29 on a basis of 70. If all students making 25 or less in 1928 had been transferred, just under thirty-three per cent would have gone back to Mathematics 4; just over thirty per cent made a grade of 25 or less in 1929.

Now taking into account all the numerous factors which tend to affect the results from such an examination—variation in time elapsed between high school and college, variation in effectiveness of the college teaching during the review, variation in student reaction to an examination, and so on—is it not clear that an appreciable proportion of the high school courses in trigonometry are poorly given as compared with the work expected of the college freshman in that subject? The conviction that such is the case grows on one when year after year he has students tell him that they did nothing beyond the solution of right triangles, or that the instructor had no particular training in mathematics, or that the course was practically turned into a course in surveying, and so on and so on.

*The high school course.*—What then is a reasonably complete course

in high school trigonometry to include? To my mind the answer is simple: Precisely the equivalent of a two-hour college course for one semester or a little more.

If this should seem somewhat ambitious, let us examine the matter from three different points of view. First let us make some comparisons of the high school and the college course in trigonometry. On examination of the catalogs of various colleges and universities in Illinois it appears that the most common course in trigonometry is a three hour course with a prerequisite of one and one-half units of algebra and one unit of plane geometry. Let us make our comparison with this course.

The high school course in trigonometry has precisely the same prerequisites as the college course; that is, the mathematical maturity required is the same in both. In a high school in which trigonometry is given during the second semester of the senior year the high school student may have a year more of work in mathematics than required for the college course. The high school senior is about a year younger on the average than the college freshman. The high school senior takes the course with about a year less of elapsed time between mathematics courses than the college freshman. The high school course meets 200 minutes per week as against 150 minutes per week in the college course, the semester being of about the same length in both. Only those who have some aptitude and liking for mathematics will elect to take trigonometry in high school, while it is required in several college curricula or the freshman may be practically forced into the course to meet group requirements.

Thus it would appear that in everything except general maturity of the students the high school has as good an opportunity as the college, if not better, in the presentation of plane trigonometry.

Again let us look at the matter from a second point of view. I believe the practice of equating two years of high school language to a year of college language is well-nigh universal. Thus a student who has taken two years of high school French, for example, will enter second year French in the university or college. If this same measuring stick be applied to high school trigonometry it should be the equivalent of two and one half hours of college trigonometry.

A third thing which bears on this question is that at least nine high schools in the state are accredited to the University of Illinois for trigonometry. That is, a student from such a high school who presents fifteen acceptable entrance units exclusive of trigonometry will

receive two hours of college credit for his trigonometry without examination. I might add that eight of these schools are also accredited for college algebra—the remaining one does not offer the course.

Then from three different standpoints it seems that my contention that the high school course in trigonometry should be the equivalent of at least a two hour college course is reasonable.

*Conclusion.*—I shall assume that trigonometry will continue to occupy a place in the curriculum of a considerable proportion of the high schools. It remains for us to inquire whether or not there are any reasons why the student should not take trigonometry in high school and repeat it in college. To my mind there are at least two good reasons why such duplication is disadvantageous.

In the first place, it requires very little thought to see that there is duplication of effort and of expenditure. The University of Illinois alone must have at least two hundred freshmen every year who repeat the first course in trigonometry after having had it in high school. Allowing twenty-five to a section, this means sixteen hours of teaching a week which is a duplication of high school work. Multiply by an appropriate factor to take care of the other universities, the five teachers colleges, and the twenty-odd endowed colleges in Illinois and the waste becomes considerable. If this is the situation in Illinois, consider the case in the country as a whole. I hazard the guess that at least 800 hours per week of instruction for one semester is spent in this duplicate instruction in this country. This figure could be materially reduced if the high school course in trigonometry were given well uniformly.

On the other hand, to start a freshman off over precisely the same material which he covered in high school the previous semester seems to me to be exceedingly poor pedagogy. The student begins his college work by developing a habit of sketchy preparation which affects directly the work in sequence and all too frequently this habit is carried over into his other work, affecting his whole college course. Every teacher of college freshmen in mathematics has watched just this happen.

It is my conviction that the high school should do thorough work in the courses which it undertakes to give. If the high school class in trigonometry is to meet five days a week for a semester and cover only about the equivalent of one hour of college trigonometry, in my judgment the high school might better spend its energies and funds in some other direction.